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MATHIEU-GROUP COVERINGS OF THE AFFINE LINE

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1. Introduction. Let L_k be the affine line over an algebraically closed field k of characteristic $p \neq 0$. Let $\pi_A(L_k)$ be the algebraic fundamental group of L_k and let $Q(p)$ be the set of all quasi p-groups, where we recall that $\pi_A(L_k)$ is defined to be the set of all finite Galois groups of unramified coverings of L_k, and a quasi p-group is a finite group which is generated by all its p-Sylow subgroups. Let a and b be nonzero elements in k and let n, t, s be positive integers such that $t < n$ and $\mathrm{GCD}(n, t) = 1$.

In [A1] it was conjectured that $\pi_A(L_k) = Q(p)$, and in support of this conjecture the unramified covering $\bar{F}_{n,q,s,a} = 0$ of L_k was considered where q is a positive power of p and

$$\bar{F}_{n,q,s,a} = Y^n - aX^sY^t + 1 \qquad \text{with } n = q + t.$$

In [A2], [A3], [AOS], and [AY], the Galois group $\bar{G}_{n,q,s,a} = \mathrm{Gal}(\bar{F}_{n,q,s,a}, k(X))$ was computed for various values of n, t, q, and this was summarized in (6.1) to (6.7) of [A4]. In the present paper we shall deal with one more case by proving the following claim.

(1.1) CLAIM. $n = 11$ *and* $t = 2$ (*and* $q = 9$ *and* $p = 3$) $\Rightarrow \bar{G}_{n,q,s,a} = M_{11}$ *where* M_{11} *is the sharply* 4*-transitive permutation group of degree* 11 *discovered by Mathieu* [M] *in* 1861. (*Note that now* $\bar{F}_{n,q,s,a} = Y^{11} - aX^sY^2 + 1$.)

By throwing away a root of $\bar{F}_{n,q,s,a}$ and deforming things suitably, we get the monic polynomial $\bar{F}'_{n,q,s,a,b,u}$ of degree $n - 1$ in Y with coefficients in $k(X)$ given by

$$\bar{F}'_{n,q,s,a,b,u} = t^{-2}[(Y + t)^t - Y^t](Y + b)^q - aX^{-s}Y^u$$

$$\text{with positive integer } u < n - 1$$

where again $n = q + t$. In [A2], [A3], and [AOS], for many values of n, t, q giving an unramified covering $\bar{F}'_{n,q,s,a,b,u} = 0$ of L_k, the Galois group $\bar{G}'_{n,q,s,a,b,u} = \mathrm{Gal}(\bar{F}'_{n,q,s,a,b,u}, k(X))$ was calculated, and this was summarized in (6.1′) to (6.7′) of [A4]. In this paper, as a consequence of (1.1), we shall calculate one more case by proving the following claim.

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