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POWER SERIES WITH HADAMARD GAPS AND HYPERBOLIC COMPLETE MINIMAL SURFACES

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1. Introduction. Let $\mathbb{D}$ be the unit disc in the complex plane $\mathbb{C}$. In this paper we are concerned with complete minimal surfaces parametrized by $\mathbb{D}$.

The literature on minimal surfaces is vast, but examples of hyperbolic complete minimal surfaces are somewhat rare, although it is well known that there is a wealth of them.

In the first part of this work we use a result of Gnuschke and Pommerenke (Theorem 1 of [2]) to exhibit a large class of hyperbolic complete minimal surfaces, thus filling part of the need for examples of these objects.

F. Xavier and L. P. M. Jorge proved in [4] the existence of complete minimal surfaces between two parallel planes in $\mathbb{R}^3$; however, their clever proof does not give any clue on how to construct examples. Here, we construct a large family of complete minimal surfaces between two parallel planes in $\mathbb{R}^3$; to do this we use some special power series with Hadamard gaps. For information on lacunary power series one can consult [1].

One of the central problems in the theory of complete minimal surfaces is to determine which meromorphic functions g can arise as Gauss maps of these surfaces. For a simply connected, hyperbolic complete minimal surface M, A. Weitsman and F. Xavier proved in [3] that g cannot be normal with order of normality less than $\sqrt{2}/2$, that g cannot have bounded characteristic, and that g cannot be a Bloch function. Here, Corollary 1 provides examples of hyperbolic complete minimal surfaces whose Gauss maps g are derivatives of analytic functions in $\mathbb{D}$ continuous up to the boundary of $\mathbb{D}$.

Let M be a simply connected minimal surface immersed in $\mathbb{R}^3$. It is well known that M can be parametrized by a pair (f, g), where f is holomorphic, g is meromorphic, z_0 is a pole of order k of g if and only if z_0 is a zero of order $2k$ of f, and f and g have as common domain $\mathbb{D}$ or $\mathbb{C}$.

The Weierstrass representation of M in terms of f and g is given by

$$
\begin{aligned}
x_1(z) &= Re \int^z \frac{1}{2} f(1 - g^2)\, dz, \\
x_2(z) &= Re \int^z \frac{i}{2} f(1 + g^2)\, dz, \qquad (1) \\
x_3(z) &= Re \int^z fg\, dz,
\end{aligned}
$$

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