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LIMITS OF SOLITON SOLUTIONS

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1. Introduction. Our main concern in this paper is a detailed study of N-soliton solutions $V_N(t, x)$ of the Korteweg-de Vries (KdV) equation in the limit $N \to \infty$. In particular, if $V_\infty(t, x)$ denotes that limit of $V_N(t, x)$ as $N \to \infty$ (in a sense to be made precise below) we shall undertake a careful investigation of the spectral (and scattering) properties of the associated Schrödinger operator $H_\infty(t) = -(d^2/dx^2) + V_\infty(t, .)$ in $L^2(\mathbb{R})$.

In order to describe our approach in more detail, we briefly recall some of the basic facts of N-soliton solutions $V_N(t, x)$ of the KdV-equation. They can be represented as

$$V_N(t, x) = -2\partial_x^2 \ln[\tau_N(t, x)], \qquad (t, x) \in \mathbb{R}^2, \tag{1.1}$$

where τ_N denotes the $N \times N$ determinant

$$\begin{gathered}\tau_N(t, x) = \det[1_N + C_N(t, x)], \\ C_N(t, x) = [c_j c_\ell (\kappa_j + \kappa_\ell)^{-1} e^{4(\kappa_j^3 + \kappa_\ell^3)t - (\kappa_j + \kappa_\ell)x}]_{j,\ell=1}^N, \\ c_j > 0, \kappa_j > 0, 1 \leqslant j \leqslant N, N \in \mathbb{N},\end{gathered} \tag{1.2}$$

and they fulfill the KdV-equation

$$\mathrm{KdV}(V_N) = V_{N,t} - 6V_N V_{N,x} + V_{N,xxx} = 0. \tag{1.3}$$

The corresponding self-adjoint Schrödinger operator $H_N(t)$ in $L^2(\mathbb{R})$ defined by

$$H_N(t) = -\frac{d^2}{dx^2} + V_N(t, .), \qquad \mathscr{D}(H_N(t)) = H^2(\mathbb{R}), \qquad t \in \mathbb{R} \tag{1.4}$$

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