Vol. 66, No. 3 DUKE MATHEMATICAL JOURNAL © June 1992

OPERATORS ON WEIGHTED BERGMAN SPACES ($0 < p \leqslant 1$) AND APPLICATIONS

OSCAR BLASCO

Introduction. During the last decade, a big effort has been made to understand operators acting on Bergman and weighted Bergman spaces. (See [A], [Z].) Different techniques have been developed for the study of different types of operators. (See [AFP], [J2] for Hankel operators, [MS] for composition operators, [W] for multipliers, ...)

The aim of this paper is to deal with operators acting on weighted Bergman spaces in the case $0 < p \leqslant 1$ and for rather general weight functions. We shall show that in this case the boundedness of an operator into a general Banach space depends only upon the behaviour of a single vector-valued analytic function. This will allow us to study Hankel operators, composition operators, and multipliers acting on weighted Bergman spaces when $0 < p \leqslant 1$ from a unified and simple technique.

The vector-valued function which represents a bounded operator is obtained by the action of the operator on the reproducing kernel. This has been previously used by N. Kalton (see [K1], [K2]) to characterize operators acting on H^p $(0 < p < 1)$ and related spaces into general q-Banach spaces and by the author (see [B]) to represent general operators acting on certain spaces of vector-valued analytic functions.

We shall be concerned with weighted Bergman classes defined by weight functions of the type introduced by S. Janson (see [J1]) which will allow us to include the known cases and to cover new ones under the same scope.

Let ρ be a nondecreasing function on (0, 1) with $\rho(0^+) = 0$ and such that $\frac{\rho(t)}{t} \in L^1((0, 1))$. Now ρ is said to be a *Dini weight* if $\int_0^s \frac{\rho(t)}{t}\,dt \leqslant C\rho(s)$. For $0 < q < \infty$, ρ is said to be a b_q*-weight*, $\rho \in b_q$, if $\int_s^1 \frac{\rho(t)}{t^{q+1}}\,dt \leqslant C\frac{\rho(s)}{s^q}$.

We say that an analytic function f on the unit disc belongs to $B_p(\rho)$, $0 < p \leqslant 1$, if

$$\|f\|_{p,\rho} = \left(\int_D \frac{\rho(1-|z|)}{(1-|z|)}|f(z)|^p\,dA(z)\right)^{1/p} < \infty.$$

For certain weights, the spaces $B_p(\rho)$ have been extensively studied in the literature. They can be regarded as extensions of the classical Bergman spaces ($\rho(t) = t$). Although the condition appearing in the case $p = 1$ and $\rho(t) = t^{1/q-1}$ for $q < 1$ goes

Received 15 August 1991. Revision received 10 October 1991.
Author partially supported by the grant C.A.I.C.Y.T. PB89-0106.