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HAUSDORFF DIMENSION OF THE SET OF NONERGODIC FOLIATIONS OF A QUADRATIC DIFFERENTIAL

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0. Introduction. Suppose X is a compact Riemann surface and q is a meromorphic quadratic differential on X. This means to each local holomorphic coordinate z is assigned a meromorphic function $q_z(z)$ such that, if w is a holomorphic coordinate in an overlapping coordinate chart,

$$q_w(w)\left(\frac{dw}{dz}\right)^2 = q_z(z).$$

Away from the zero and poles of q, there are "natural coordinates" z such that $q_z(z) \equiv 1$. We will assume the poles, if any, are simple. This is equivalent to saying

$$\int_X |q(z)\, dz^2| < \infty.$$

We will consider the 1-parameter family of quadratic differentials $e^{2i\theta}q$ as θ varies in the interval $[-\pi/2, \pi/2]$. Natural objects of study are the vertical trajectories of $e^{2i\theta}q$. These are the curves along which $e^{2i\theta}q\, dz^2 < 0$. The collection of vertical trajectories of $e^{2i\theta}q$ forms the leaves of a measured foliation F_θ. We are interested in studying the ergodic properties of the foliation F_θ as θ varies.

An important topological property of a measured foliation is minimality. A foliation is minimal if every closed set which is a union of leaves is either empty or the whole surface. A measure-theoretic analogue is ergodicity. A foliation is ergodic if every measurable set which is a union of leaves either has measure zero or its complement does. A stronger property than ergodicity is unique ergodicity. A foliation is uniquely ergodic if it has a unique, up to scalar multiple, transverse invariant measure. That the properties of minimality and unique ergodicity are different is established by examples of minimal nonuniquely ergodic measured foliations, constructed implicitly or explicitly by Veech [V1], Keynes and Newton [KN], Satayev [Sa], and Keane [K]. Accordingly, let $NM(q)$ be the set of θ such that F_θ is not minimal. Results of [S] and [ZK] show that $NM(q)$ is countable. Let $NE(q)$ denote the set of θ such that F_θ is not ergodic. Let $NUE(q) \supset NE(q)$ denote

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