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FAMILIES OF GALOIS REPRESENTATIONS —INCREASING THE RAMIFICATION

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0. Introduction. Let $\bar{\rho}$: $\mathrm{Gal}(\overline{\mathbf{Q}}/\mathbf{Q}) \to GL_2(k)$ (k a finite field of characteristic $\ell \neq 2$) be a continuous absolutely irreducible odd representation unramified outside a finite set S_1 of rational primes, where $\ell \in S_1$. Suppose $\bar{\rho}$ is associated to a modular form (as all such $\bar{\rho}$ are conjectured to be [12]) and that this modular form is of weight 2.

In this paper we study cases where increasing the set of primes allowed to be ramified in lifts of $\bar{\rho}$ to $W(k)$-algebras enlarges the space of these lifts. This paper might be compared with a paper of Ribet [9], where increasing the set of ramified primes increases the number of corresponding modular forms.

There, Ribet considers adding to S_1 a prime $p \notin S_1$ satisfying the hypothesis

$$(1) \qquad a_p \equiv \pm(1 + p) (\mathrm{mod}\ \ell)$$

where a_p is the trace of Frobenius at p of $\bar{\rho}$ (or of the corresponding modular form). He shows that under hypothesis (1) there is a "p-new" form also associated to $\bar{\rho}$.

We make the further assumption

$$(2) \qquad p \not\equiv 1 (\mathrm{mod}\ \ell)$$

and show that under hypotheses (1) and (2) the space of Galois representations lifting $\bar{\rho}$ increases in size if ramification is allowed at p in addition to S_1. If $p \not\equiv -1$ (mod ℓ), this space "doubles". If $p \equiv -1$ (mod ℓ), then this space "increases $(\ell^r + 5)/2$-fold", where ℓ^r is the exact power of ℓ dividing $p + 1$.

Ribet's p-new forms are necessarily in these mysterious new spaces of representations. One new feature is that, in case (b) below, whereas Ribet's method produces 2 new modular forms, our method adds $(\ell^r + 3)/2$ new parameters to the space of representations, which suggests that there may be yet more modular forms, congruent to our original one, to be found. Ribet has indicated that these modular forms can indeed be obtained using ideas of Carayol.

I thank Carayol for explaining how these arise. See [2] for more details. Let X be a Shimura curve over $\mathbf{Q}$. Let Γ be an abelian group acting on X. Then $H^1(X_{\overline{\mathbf{Q}}}, \mathbf{Q}_\ell) = \bigoplus_i V_i$, where the V_i's are two-dimensional Γ-modules. Let the char-

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