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ANALOGUES OF THE BRAUER GROUP FOR ALGEBRAS WITH INVOLUTION

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Introduction. Let X be a scheme with $1/2 \in \Gamma(X, \mathcal{O}_X)$. We define a notion of equivalence between Azumaya algebras with involution on X, analogous to that used in defining the Brauer group, where the trivial equivalence class consists of endomorphism algebras of locally free sheaves on X with involutions induced by nondegenerate symmetric bilinear forms. Let $\mathrm{Br}^*(X)$ be the group of such equivalence classes of algebras with involution. We show (see Theorem 1) that there is a natural injection

$$\mathrm{Br}^*(X) \to H^0_{\text{ét}}(X, \mu_2) \oplus H^2_{\text{ét}}(X, \mu_2),$$

which is an isomorphism precisely if the 2-torsion in the cohomological Brauer group $H^2_{\text{ét}}(X, \mathbf{G}_m)$ is represented by classes of Azumaya algebras.

Next, suppose $\pi: Y \to X$ is an étale cover of degree 2 of schemes (on which 2 is invertible). Let δ be the nontrivial automorphism of Y/X. We define an analogous group $\mathrm{Br}(X, \delta)$ of equivalence classes of Azumaya algebras Y with an involution of the second kind (one which acts by δ on the centre). Here, the trivial class consists of endomorphism algebras of locally free $\mathcal{O}_Y$-modules with involutions induced by nondegenerate δ-Hermitian forms. If $\mathcal{G}$ is the étale sheaf on X obtained from invertible functions on Y of norm 1, then there is a natural injection

$$\mathrm{Br}(X, \delta) \to H^2_{\text{ét}}(X, \mathcal{G}),$$

which is an isomorphism precisely if every class in

$$\ker(N_{Y/X}: H^2_{\text{ét}}(Y, \mathbf{G}_m) \to H^2_{\text{ét}}(X, \mathbf{G}_m))$$

is represented by an Azumaya algebra on Y. There is an associated exact sequence

$$\mathrm{Pic}\, Y \xrightarrow{N_{Y/X}} \mathrm{Pic}\, X \to \mathrm{Br}(X, \delta) \to \mathrm{Br}(Y) \xrightarrow{N_{Y/X}} \mathrm{Br}(X).$$

If $(\mathcal{E}, q)$ is a quadratic space of even rank on a scheme X, one associates to it the Clifford invariant, i.e., the class of its Clifford algebra $C(q)$ in ${}_2\mathrm{Br}(X)$. The algebra $C(q)$ comes equipped with two involutions defined in a natural way, such that the corresponding classes in $\mathrm{Br}^*(X)$ have the same component in $H^2_{\text{ét}}(X, \mu_2)$,

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