Vol. 65, No. 3 DUKE MATHEMATICAL JOURNAL © March 1991

RIEMANNIAN MANIFOLDS WITH SMALL INTEGRAL NORM OF CURVATURE

DEANE YANG

J. Cheeger and M. Gromov [4] proved that, if a manifold has a Riemannian metric such that the injectivity radius is sufficiently small everywhere relative to the sectional curvature, then the manifold admits an F-structure of positive rank and collapses with bounded curvature. (See [5, 4] for the definition and examples.) More recently, in joint work with K. Fukaya, they have refined the notion of F-structures into what they call N-structures. Gromov has asked if a compact n-dimensional Riemannian manifold with sufficiently small $L^{n/2}$ norm of curvature must admit an F-structure of positive rank.

Gromov's question is an important one. Cheeger, Gromov, and others have extensively studied how pointwise bounds on curvature control the geometry and topology of a Riemannian manifold. Given a sequence of metrics with uniform pointwise bounds on curvature, the Cheeger-Gromov(-Greene-Wu-Peters) convergence theorems [3, 8, 10, 11, 12] show that, when the injectivity radius is bounded from below, there is a convergent subsequence. On the other hand, the Cheeger-Gromov collapsing manifold theorem [5, 4] describes the topology of a manifold that admits a metric with bounded sectional curvature and sufficiently small injectivity radius everywhere. Analogous results using integral bounds on curvature would open up a whole new range of possibilities for using analysis to study the topology of a manifold.

Two metrics are C^0 *close* if the distance between them is less than 1 with respect to the C^0 topology.

The purpose of this paper is to prove the following result.

THEOREM 0.1. *Given* $3 \leqslant n < q$ *and* $c > 0$, *there exist constants* $\delta(n, c)$, $\kappa(n, q, c) > 0$, *and* $C(n, q, c) > 0$ *such that the following holds.*

Given $0 < \varepsilon < \kappa(n, q, c)^2$ *and a smooth n-dimensional manifold M with a complete Riemannian metric g satisfying*

$$\|\rho(\delta, \cdot)\|_\infty^{q-n} \int_M |\mathrm{Rc}|^{q/2}\, dV \leqslant \kappa(n, q, c)^q$$

where ρ *is the weak injectivity radius (see* §1 *for the definition) and*

$$\int_M |\mathrm{Rm}|^{n/2}\, dV \leqslant \varepsilon^{n/2},$$

Received 23 August 1991.