Vol. 65, No. 1   DUKE MATHEMATICAL JOURNAL ©   January 1992

THE SPECTRUM OF THE LAPLACIAN OF MANIFOLDS OF POSITIVE CURVATURE

JOSE F. ESCOBAR AND ALEXANDRE FREIRE

Introduction. The Laplace-Beltrami operator Δ of a complete noncompact Riemannian manifold is an unbounded, essentially selfadjoint operator, and the spectrum $\sigma(\Delta)$ of its selfadjoint extension to L^2 functions is an important geometric invariant. By the spectral theorem, the closed set $\sigma(\Delta)$, the spectral measure class, and the multiplicity function class classify Δ as a selfadjoint operator in L^2, up to unitary equivalence. One of the main results of this paper is the determination of $\sigma(-\Delta)$ as a set for manifolds with an end of nonnegative curvature whose Ricci curvature satisfies an integrability condition. (See Theorem 2 below for the precise statement.) Regarding the spectral measure, the first question that arises is whether, for a given class of complete noncompact Riemannian manifolds, the point spectrum is nonempty, or equivalently, whether Δ has eigenfunctions in L^2. For Schrödinger operators $-\Delta + V$ in $\mathbb{R}^n$, conditions are known under which there are no eigenstates of positive energy [RS]. For complete manifolds, although examples are known where one finds eigenvalues embedded in the essential spectrum [Don], it is expected that this cannot happen under appropriate curvature conditions. In particular, it has been conjectured that for complete manifolds of nonnegative curvature there should be no eigenvalues for Δ. In this paper we verify this under an additional condition. Recall that a point p_0 in a Riemannian manifold is a *pole* if the exponential map $\exp_{p_0}: T_{p_0}M \to M$ is a diffeomorphism.

THEOREM 1. *Let M be a (complete) n-dimensional manifold with a pole and with nonnegative sectional curvatures K_r in radial directions. Assume either*
(i) *$n = 2$, or*
(ii) *$n \geqslant 3$ and $0 \leqslant K_r \leqslant c_n(1 - c_n)/r^2$ for all $r > 0$, where $c_n = (n - 2)/n$ and r denotes the distance to the pole.*
Then there are no eigenfunctions in $L^2(M)$.

Previously, this result had been obtained by the first author [Esc] under the assumption that, outside of a compact set, the metric of M is rotationally symmetric and nonnegatively curved, and with no decay conditions on the curvature. The main ingredient in our proof of triviality of the point spectrum is an integral identity for eigenfunctions. Although this particular formula seems not to have been observed previously, similar identities were obtained classically by F. Rellich [Rel] and more recently and in a similar context by F. Tayoshi [Tay], F. Xavier [Xav] and N. Garofalo-F. H. Lin [GL].

Received 7 June 1991. Revision received 27 June 1991.