Vol. 64, No. 2 DUKE MATHEMATICAL JOURNAL © November 1991

LOCAL ANALYTICITY FOR THE $\bar{\partial}$-NEUMANN PROBLEM AND $\square_b$—SOME MODEL DOMAINS WITHOUT MAXIMAL ESTIMATES

MAKHLOUF DERRIDJ AND DAVID S. TARTAKOFF

Introduction. We continue the study of local real analytic regularity (up to the boundary) for solutions to the $\bar{\partial}$-Neumann problem and real analytic hypoellipticity for $\square_b$. For a detailed history of regularity results for these problems, going back to the work of Kohn in 1963 ([Koh1, Koh2]), the reader may refer to [DT3, DT4].

In recent papers ([DT3, DT4]) we proved local real analytic hypoellipticity (at the origin) in both the $\bar{\partial}$-Neumann problem and $\square_b$ for pseudoconvex domains which are (locally) of the form

$$\operatorname{Im} w > h(|z|^2) \qquad (z \in \mathbb{C}^n) \tag{0.1}$$

with $h(s)$ real analytic (and not constant), $h(0) = 0$. We used purely L^2-methods generalizing those of Tartakoff ([T4, T5]) and used results of Derridj and Grigis-Rothschild ([D2], [GR1]) which show that one has "maximal" estimates for such domains. We also needed to construct a special holomorphic vector field M and then localize high powers of M in place of the localizations of high powers of $T = \partial/\partial t$, $t = \operatorname{Re} w$, of [T4, T5].

If we want to consider more general domains such as

$$\operatorname{Im} w > h_1(|z'|^2) + h_2(|z''|^2) \tag{0.2}$$

and try to use the main ideas of the proof of [DT3, DT4], we must of course first find an effective substitute for the "maximal estimate". But, in addition, in using cutoff functions as in our earlier work, we arrive at derivatives of these cutoff functions with respect to z' and z'' which are nonzero in regions where the Levi form degenerates, and we cannot apply the local results of [T4] and [Tr1] in such regions. Instead, we must mix the methods of our recent joint papers and the methods of [T4, T5] so that, in the region where derivatives of the cutoff functions are nonzero, we may utilize our recent results and techniques. For the present we shall be able to handle pseudoconvex domains (and their boundaries) of the form

$$\operatorname{Im} w > g(z, \bar{z}) = \sum_{j=1}^{p} |z_j|^2 + h\left(\sum_{j=p+1}^{n} |z_j|^2\right) = |z'|^2 + h(|z''|^2) \tag{0.3}$$

with h as in (0.1).

Received 20 August 1990. Revision received 15 February 1991.
Tartakoff partly supported under NSF grant DMS 88-01218.