Vol. 62, No. 3 DUKE MATHEMATICAL JOURNAL © April 1991

EFFECTIVE p-ADIC BOUNDS AT REGULAR SINGULAR POINTS

G. CHRISTOL AND B. DWORK

Introduction. Let k be an algebraically closed field of characteristic zero which is complete under a nonarchimedean valuation and has residue class field of characteristic p and rank one valuation group.

The gauss norm on $k[x]$ is the supremum of the magnitudes of the coefficients. This norm is extended to $k(x)$ in the obvious way. Let E be the completion of $k(x)$ under this norm. Let $E_0 = \{\xi \in E \mid \xi$ is analytic function on $D(0, 1^-)$, the open disk of radius unity and center $0\}$. Let E_0' be the quotient field of E_0. Let $\delta = x(d/dx) = xD$.

Finite dimensional spaces over E with a natural basis are given a norm derived from E in the standard way. This applies to n-tuples, to matrices and to exterior products of n-tuples.

In particular an element of $G\ell(n, E)$ will be said to be unimodular if it and its inverse are bounded by unity.

For $G \in \mathscr{M}_n(E)$, $H \in G\ell(n, E)$ we define

$$G_{[H]} = \delta H \cdot H^{-1} + HGH^{-1}.$$

As is well known, if $\delta y = Gy$ for some n-tuple y with coefficients in a differential field extension of E, then $z = Hy$ implies $\delta z = G_{[H]}z$.

For $u \in E$ were define u^ϕ by the composition $x \mapsto u(x^p)$.

We view elements of $k(x)$ (and of E) as being functions on subsets of a universal domain Ω containing k to which the valuation of k is extended. In particular we may assume that the residue class field of Ω is transcendental over that of k. Let $t \in \Omega$ be a generic unit in this sense.

Let $\mathscr{R}$ be the set of all $G \in \mathscr{M}_n(E)$ satisfying the following four conditions:

$\mathscr{R}1$. $G \in \mathscr{M}_n(E_0)$.

$\mathscr{R}2$. The equation $(\delta - G)y = 0$ has a solution matrix $\mathscr{U}_{G,t}$ at t (normalized by $\mathscr{U}_{G,t}(t) = I$) which converges on $D(t, 1^-)$.

$\mathscr{R}3$. $G(0)$ is nilpotent.

$\mathscr{R}4$. $|G|_E \leqslant 1$.

Condition $\mathscr{R}3$ implies that equation $\delta - G$ has a solution matrix at 0 which may be written uniquely in the form $Y_G x^{G(0)}$, where

$$Y_G \in G\ell(n, k[[x]])$$

$$Y_G(0) = I_n.$$

Received 27 January 1990.