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ON HIGHER WEIERSTRASS POINTS

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This paper deals with flexes of the n-canonical linear series, or n-Weierstrass points, on a compact Riemann surface.

Based on the theory of Limit Linear Series of Eisenbud and Harris we identify the limit of the n-canonical linear series, and hence the limits of its flexes, as a smooth curve degenerates to a stable curve of compact type. This allows us to prove some propositions that have known analogues in the case of 1-Weierstrass points.

After some preliminaries in the first four sections, we consider in §5 the "Hurwitz problem": which sequences occur as vanishing sequences of n-Weierstrass points? By a regeneration argument we prove (Theorem 5.3) that every sequence of low enough weight occurs in the right dimension. Also, we reprove Lax's result [L], Theorem 3, to the effect that a general curve of genus $g \geqslant 3$ has only ordinary n-Weierstrass points.

In §6 we prove that the hypersurfaces $\mathscr{W}_g^n$ of n-Weierstrass points in the moduli spaces of pointed curves are irreducible; see Theorem 6.1 for a more precise statement. We do this by showing that the monodromy of $\mathscr{W}_g^n \to \mathscr{M}_g$ is transitive; the basic idea to generate monodromy being as in [E-H, 3], namely, to vary the limit n-canonical series of a fixed curve.

In the last section we compute the linear equivalence class of the closure of $\mathscr{W}_g^n$ in the moduli space of stable pointed curves. This adds some more members to our collection of effective divisor classes on moduli space (see Remark 7.8).

CONTENTS

§1. Flexes. Let X be a smooth irreducible complete algebraic curve over the complex numbers, and L a g_d^r on X (that is, $L = (V, \mathscr{L})$ where $\mathscr{L}$ is a line bundle on X of degree d and V is a linear space of sections of $\mathscr{L}$, of dimension $r + 1$).

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