Vol. 62, No. 1 DUKE MATHEMATICAL JOURNAL © February 1991

AFFINE GRASSMANNIAN HOMOLOGY AND THE HOMOLOGY OF GENERAL LINEAR GROUPS

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1. Introduction. For more than two decades algebraic geometrists have been looking for a cohomology theory defined at least for quasi-projective schemes over some base scheme S often refered to as Motivic Cohomology. More recently, Beilinson conjectured that Motivic Cohomology might be taken as the hyper-cohomology of certain sheaves of complexes, which should all be defined functorially on quasi-projective schemes subject to a certain set of axioms. Since then various candidates for such complexes were discovered. Among them is Bloch's complex [1], which computes the so-called higher Chow groups, as well as a linear version of it, the Grassmannian complex of [2], which gives the so-called Grassmannian Homology groups.

The paper is organized as follows.

Section 1 replaces the Grassmannian Homology complex of [2] by a different (affine) version to relate the Grassmannian Homology groups more naturally to Bloch's higher Chow groups. The motivation to study those Grassmannian Homology groups originated from a conjectured isomorphism between them and Bloch's higher Chow groups speculated in [2]. In fact the affine Grassmannian Homology complex is a subcomplex of Bloch's complex $z(Speck, *)$ [1], but unfortunately, the inclusion of complexes does not induce an isomorphism of homology groups. We will show in a later article [4] that the Grassmannian Homology groups are too large as it can be demonstrated on the indecomposable part $K_3(k)$. However, the Grassmannian Homology groups are still tied to algebraic K-theory via their relation to the homology of general linear groups. Namely, we have

THEOREM 3.5. *There is an isomorphism*

$$^{A}\mathrm{GH}^{p}_{q} \cong H_{p+q}(cone(^{H}\mathrm{CG}^{p}_{*} \xrightarrow{i} {}^{P}\mathrm{CG}^{p}_{*})) \cong H_{p+q}(GL_p(k), GL_{p-1}(k)).$$

In 4.3 we also show that this isomorphism is induced by the Chern class map ch^p of [2] after extending its construction to the case of the relative homology of the pair (GL_p, GL_{p-1}).

Acknowledgements. This paper is part of my thesis written at MIT. I would like to thank my advisor Bob MacPherson for many inspiring and stimulating discussions and for his steady encouragement.

Received 20 December 1989.