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SMALL POINTS ON CONSTANT ARITHMETIC SURFACES

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In Faltings [3], it is shown that the relative dualizing sheaf of a semistable arithmetic surface has nonnegative self-intersection with respect to the Arakelov metric. Many authors have raised the question of when $\omega_{X/S}^2 = 0$ can occur. Szpiro [10], while believing it to be an unlikely phenomenon, has suggested the appellation "*constant*" *arithmetic surfaces* for those with this property. In this note, as a converse to Szpiro [10], we use Raynaud's theorem on torsion points and embedded curves [9], together with the technique in Kim [7] to derive a rather curious consequence of $\omega_{X/S}^2 = 0$ in the case of a *smooth* arithmetic surface. The Jacobian of such a surface is shown to possess an infinite sequence of distinct, nontorsion, algebraic points, lying inside an image of the curve, whose canonical heights tend to zero.

This note is obviously just an appendix to Szpiro [10], to whose other diverse writings on Arakelov Theory I am equally indebted. The proof of Theorem 2 also benefited greatly from a lecture of G. Faltings.

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1. Let X be an arithmetic surface over the ring of integers R in a number field F; i.e., X is a scheme, projective and flat over $S = Spec(R)$ of relative dimension one. We further assume that the generic fiber X_F is smooth and geometrically irreducible of genus $g \geqslant 1$, and that X is regular.

For each archimedean place v of F, choose an embedding $\sigma: F \to \mathbf{C}$ associated to v and denote by X_v the analytic space associated to the complex points of $X_F \times_F \mathbf{C}_\sigma$. As in Faltings [3], we can choose a basis $\{\omega_1, \omega_2, \ldots, \omega_g\}$ of holomorphic differentials on X_v orthonormal with respect to the inner product

$$(\omega_i, \omega_j) = \frac{\sqrt{-1}}{2} \int_{X_v} \omega_i \wedge \bar{\omega}_j \tag{1}$$

and define

$$\mu_v = \frac{\sqrt{-1}}{2g} \sum_{i=1}^{g} \omega_i \wedge \bar{\omega}_i. \tag{2}$$

This clearly provides a probability measure for X_v.

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