Vol. 60, No. 3 DUKE MATHEMATICAL JOURNAL © June 1990

BIFURCATION FROM INFINITY IN A NONLINEAR ELLIPTIC EQUATION INVOLVING THE LIMITING SOBOLEV EXPONENT

OLIVIER REY

1. Introduction. Let Ω be a smooth and bounded domain in $\mathbb{R}^3$, and consider the problem

$$(I) \qquad \begin{cases} -\Delta u = u^5 + \mu u^q, u > 0 & \text{on } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

where $q \in]2, 3[$ and $\mu \in \mathbb{R}^*$.

We will prove in this paper two distinct results on the above equation.

(a) The first result concerns the multiplicity of solutions of (I), for μ large. Brézis and Nirenberg have shown in [5] that for $q \in]1, 3]$, there exists $\mu_0 \geqslant 0$ (which depends on q and Ω) such that for any $\mu \geqslant \mu_0$ (I) has at least one solution. Moreover, they suggested that the following holds: if $q \in]1, 3[$, there exists some $\mu_0 > 0$ such that for any $\mu > \mu_0$ (I) has at least two solutions. Atkinson and Peletier proved in [1] that this is true when Ω is a ball. The following result provides us with another partial answer to the conjecture:

THEOREM 1. *Let Ω be any smooth and bounded domain in $\mathbb{R}^3$, of Ljusternik-Schnirelman category p; let $q \in]2, 3[$. For $\mu > 0$ large enough, there are at least $p + 1$ solutions of (I).*

(b) The second result concerns a phenomenon of bifurcation which may occur under a continuous deformation of the domain Ω.

Let us recall some facts; if Q is a smooth and bounded domain in $\mathbb{R}^N$, $N \geqslant 4$, and (P_ε) denotes the problem:

$$(P_\varepsilon) \qquad \begin{cases} -\Delta u = u^{(N+2)/(N-2)} + \varepsilon u, u > 0 & \text{on } Q \\ u = 0 & \text{on } \partial Q \end{cases}$$

it has been shown in [6] [7] that there exist branches of solutions u_ε, for $\varepsilon > 0$ small enough, which concentrate at distinguished points of Q when ε tends to zero; namely

$$|\nabla u_\varepsilon|^2 \xrightarrow[\varepsilon\to 0]{} \bar{S}\delta_{x_0} \quad \text{in the sense of measures.}$$

$$u_\varepsilon^{2N/(N-2)} \xrightarrow[\varepsilon\to 0]{} \bar{S}\delta_{x_0} \quad \text{in the sense of measures.}$$

Received November 16, 1989.