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A GEOMETRIC CONSTRUCTION OF A RESOLUTION OF THE FUNDAMENTAL SERIES

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Introduction. Since the fundamental work of Harish-Chandra, the discrete series have played an important role in the representation theory of real semisimple groups. The problem of realization of the discrete series apart from its own importance very often served as a source of inspiration for the construction of new representations. Various realizations of the discrete series are known, for instance, a geometric one by Schmid and algebraic one by Enright and Varadarajan. In the case when a real semisimple group does not have a compact Cartan subgroup the fundamental series appear as closest analogues of the discrete series representations. In fact, they are induced from the discrete series of Levi subgroups associated with maximally compact Cartan subgroups. In [4], Enright developed an infinitesimal approach to study the fundamental series. This was based on his earlier work with Varadarajan ([6]) and Wallach ([5]). The present paper is an attempt to understand Enright's construction in terms of geometry of the flag variety. Our approach is based on the theory of $\mathscr{D}$-modules and in particular on Beilinson-Bernstein localization technique. It enables us to obtain geometric realization of Enright-Varadarajan modules and to exhibit the connection of Enright's construction with the theory of highest weight modules in a quite transparent way.

Let G_0 be a connected semisimple real Lie group with finite center, K_0 a maximal compact subgroup of G_0 and θ the corresponding Cartan involution. Let $\mathfrak{g}_0$ and $\mathfrak{k}_0$ be the Lie algebras of G_0 and K_0 respectively, $\mathfrak{g}$ and $\mathfrak{k}$ their complexifications, and K the complexification of K_0. Denote by X the flag variety of Borel subalgebras in $\mathfrak{g}$.

Using the Beilinson-Bernstein localization technique ([1]) we can pass from the modules over enveloping algebra $\mathscr{U}(\mathfrak{g})$ to $\mathscr{D}$-modules and conversely. The $\mathscr{D}$-modules we are interested in appear in the following way: we start with a smooth subvariety Z of X and a line bundle $\mathscr{L}$ on X. If $\mathscr{D}$ is a sheaf of differential operators on $\mathscr{L}$, then the local cohomology sheaves $\mathscr{H}_Z^i(\mathscr{L})$ carry a natural structure of $\mathscr{D}$-module. Therefore, on the corresponding local cohomology groups $H_Z^i(X, \mathscr{L})$ we obtain (in general infinite-dimensional) representations of $\mathscr{U}(\mathfrak{g})$. Moreover, if Z is invariant under the action of some subgroup $L \subset \mathrm{Int}(\mathfrak{g})$, then $H_Z^i(X, \mathscr{L})$ is naturally endowed with a structure of $(\mathscr{U}(\mathfrak{g}), L)$-module.

The action of K on X defines finitely many affinely imbedded orbits. In particular, we fix a closed K-orbit defined by a θ-stable Borel subalgebra $\mathfrak{b}$. Denote by x_0 a point in X representing $\mathfrak{b}$ and put $Y = K \cdot x_0$. Let $\mathfrak{h}$ be a θ-stable fundamental

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