Vol. 59, No. 2 DUKE MATHEMATICAL JOURNAL © October 1989

IWASAWA *L*-FUNCTIONS FOR MULTIPLICATIVE ABELIAN VARIETIES

JOHN W. JONES

Bernardi, Goldstein, and Stephens [1, 2], and subsequently Mazur, Tate, and Teitelbaum [9], have formulated p-adic analogues of Birch and Swinnerton-Dyer's conjectures for analytic p-adic L-functions of elliptic curves based on calculations of their special values and their derivatives. The calculations in [9] included curves with split multiplicative reduction at p, in which case the p-adic L-function appeared to vanish to higher order than their classical analogues.

One expects to find similar behavior between analytic p-adic L-functions and Iwasawa L-functions. Investigations of this type were initiated by Perrin-Riou [12] and generalized by Schneider [14, 15]. Their work was concerned with curves with good ordinary reduction. In this paper, we deal with the case of multiplicative reduction (thus encompassing the extra zeroes).

The Iwasawa L-function of an elliptic curve is traditionally defined as the characteristic polynomial of the p-Selmer group for the curve. We investigate this polynomial and find that while its leading coefficient has the predicted p-adic ordinal, it does not exhibit the extra zero, suggesting that it is not the correct choice of Λ-module. We suggest several possibilities for the Λ-module which do have the correct order of vanishing and first nonvanishing coefficient. Finally, we prove a functional equation for these modules, generalizing the function equation proved by Mazur [7] in the good ordinary reduction case.

I would like to thank my advisor, Barry Mazur, for his help and guidance.

1. Notations, conventions, and terminology. We will observe the following notations, conventions, and terminology throughout:

Let $p \neq 2$ be a prime number and let K be a number field. If F is a field, $\mathcal{O}_F$ will be its ring of integers (if appropriate).

A *quasi-isomorphism* is a group homomorphism with finite kernel and cokernel. Similarly, we will refer to *quasi-injections*, *quasi-surjections*, and *quasi-exact sequences* with the obvious meanings intended.

If G is an abelian group, we let G^* denote the Pontryagin dual of G; Tor $G :=$ the torsion subgroup of G; Div $G :=$ its subgroup of divisible elements; $G_{\mathrm{Tor}} := G/\mathrm{Tor}\ G$ and $G_{\mathrm{Div}} := G/\mathrm{Div}\ G$. Furthermore, if $f\colon A \to B$ is a homomorphism, we let Div f and f_{Div} be the induced maps Div $A \to$ Div B and $A_{\mathrm{Div}} \to B_{\mathrm{Div}}$, respectively. We

Received October 7, 1988. Revision received April 3, 1989.