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TRACE FORMULA FOR COMPACT $\Gamma\backslash PSL_2(\mathbb{R})$ AND THE EQUIDISTRIBUTION THEORY OF CLOSED GEODESICS

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0. Introduction. Let Γ be a discrete, cocompact subgroup of hyperbolic elements of $G = PSL_2(\mathbb{R})$, and let X_Γ be the associated hyperbolic surface $\Gamma\backslash G/K$. The unit (co-) tangent bundle $S^*(X_\Gamma)$ may then be identified in a familiar way with $\Gamma\backslash G$, and under this identification right translation by $a_t = \begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix}$ corresponds to the geodesic flow G^t. Further, conjugacy classes $\{\gamma\}$ of hyperbolic elements determine closed geodesics $\bar{\gamma}$ in $S^*(X_\Gamma)$ (periodic orbits of G^t). For each closed geodesic $\bar{\gamma}$, let μ_γ denote the positive measure on $C(S^*X_\Gamma)$ defined by $\langle f, d\mu_\gamma\rangle = \int_{\bar{\gamma}} f$. Then let $\mu_T = \Sigma_{L_\gamma \leqslant T}\mu_\gamma$, where L_γ is the length (or period) of $\bar{\gamma}$; equivalently, γ is conjugate to $a_{L_\gamma/2}$. The equidistribution theorem for closed geodesics on X_Γ is as follows:

THEOREM (Bowen [B]). *For any* $f \in C(S^*X_\Gamma)$, $\lim_{T\to\infty} \frac{\mu_T(f)}{\mu_T(1)} = \frac{1}{\mathrm{vol}(S^*X_\Gamma)} \times \int_{S^*X_\Gamma} f\, d\omega$, *where* $d\omega$ *is Liouville* (*or Haar*) *measure.*

In other words, periodic geodesics on compact hyperbolic surfaces become equidistributed relative to Haar measure as the period tends to ∞. (This is *not* the effect of the averaging over *all* geodesics γ of length $\leqslant T$; see the remarks at the end of §0, and §5.)

Our purpose in this paper is to sharpen Bowen's theorem by giving asymptotic expansions for $\mu_T(f)$ when f is a Casimir eigenfunction of some weight m. We will do this means of Selberg-type trace formulae. The guiding principle is this: Suppose $\sigma_{s,m}$ is an automorphic form of weight m, lying in an irreducible representation $\mathscr{H}(s)$ for G:

$$\begin{cases} \Omega\sigma_{s,m} = (s-1)(s+1)\sigma_{s,m} \\ \frac{1}{i} W_{s,m} = m\sigma_{s,m} \end{cases},$$

where Ω is the Casimir and $W \sim \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ (notation of [L]). Let γ_0 denote the generator of the centralizer Γ_γ of γ (corresponding to the primitive geodesic deter-

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