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WEIGHTED SIEVES AND TWIN PRIME TYPE EQUATIONS

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I. Presentation of the techniques. In this paper we are mainly concerned with the equation

$$p + 2 = P_2 \tag{1.1}$$

where p denotes a prime and P_k any integer with at most k prime factors. By Chen's Theorem ([3], [4]) we know that (1.1) has infinitely many solutions and in the same way, the Goldbach type equation

$$2N = p + P_2$$

is solvable when the integer N is sufficiently large. The classical proof of (1.1) consists of shifting the sequence

$$\mathscr{A} = \{p + 2; p \leqq x\}$$

up to $z = x^{1/a}$, where a denotes a sufficiently large constant. Kuhn weights are attached to the elements contributing to the shifting function

$$S(\mathscr{A}, x^{1/a}) = |\{n \in \mathscr{A}; p|n \Rightarrow p \geqq x^{1/a}\}|$$

and the end of the proof one uses Selberg's upper bound sieve, in the form given in the Jurkat-Richert Theorem.

The behavior of $\mathscr{A}$ in residue classes is controlled by the celebrated Bombieri-Vinogradov Theorem. However, it is necessary to appeal to a specific property of the sequence $\mathscr{A}$, which does not appear in the general statement of a sieve problem: more precisely, to end the proof, we need to majorize the cardinality of the subset of $\mathscr{A}$ of elements of the form $p_1p_2p_3$ where the p_i satisfy certain inequalities. Now it is easy to see that this question is reduced to bounding the number of primes of the form

$$p_1p_2p_3 - 2.$$

The rôle of the variables has been inverted: $\mathscr{A}$ satisfies the switching principle ([9]).

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