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SPECTRAL ANALYSIS OF SECOND-ORDER ELLIPTIC OPERATORS ON NONCOMPACT MANIFOLDS

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0. Introduction. When studying the spectrum of the Laplace operator on a noncompact manifold, one is immediately faced with the task of locating and describing the essential spectrum. In this paper we will prove the Mourre estimate and related bounds for a class of elliptic operators which includes Laplacians for both cofinite and coinfinite quotients of hyperbolic space. These estimates imply, via the Mourre theory, resolvent estimates which show the absence of singular continuous spectrum. The Mourre estimate also gives information about accumulation of embedded eigenvalues. In a companion paper we will study the question of existence of embedded eigenvalues and exponential decay of eigenfunctions. It is known that for manifolds which are small at infinity, embedded eigenvalues may occur. We will show that they do not occur if the manifold is large at infinity.

There is a large and growing literature on spectral geometry, i.e., the relationship between the geometry of a Riemannian manifold and the spectrum of the associated Laplace operator. An extensive bibliography can be found in Bérard [3]; see also Chavel [4]. Much of this work concerns compact manifolds for which the spectrum of the Laplacian is purely discrete. Work on the noncompact case has centered on manifolds (of constant negative curvature) which are quotients of hyperbolic space by discrete groups of isometries. The spectral theory of Laplacians on these manifolds was first studied by number theorists, e.g., Maass [13], Selberg [20], and Roelcke [19]. In [9], Faddeev introduced methods of scattering theory to obtain the spectral theory of the Laplacian on two-dimensional hyperbolic manifolds of finite area. Since then, various approaches to scattering theory have been applied to increasingly general quotients of hyperbolic space (and perturbations thereof) by Patterson [16], Lax and Phillips [10]–[12], Perry [17], Agmon [2], Phillips, Wiscott, and Woo [18], and others. More general noncompact manifolds which are not quotients of hyperbolic space but whose metrics near infinity are known fairly explicitly and have negative sectional curvature have been treated by Donnelly [7] and by Mazzeo and Melrose [14].

We were guided by recent work of Escobar [8] and DeBièvre and Hislop [6], who studied embedded eigenvalues and resonances, respectively, on manifolds for which one can separate variables. Our approach is to use elliptic second-order operators with this property near infinity as the "free Hamiltonian" in the Mourre

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