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ON CONFORMAL SCALAR CURVATURE EQUATIONS IN $\mathbb{R}^n$

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1. Introduction. On a Riemannian manifold (M, g) of dimension n, $n \geqslant 3$, the problem of finding conformal metrics with prescribed scalar curvature K is equivalent to finding positive solutions of the elliptic equation on M

$$\frac{4(n-1)}{(n-2)}\Delta_g u - ku + Ku^{n^*} = 0, \tag{1.1}$$

where $n^* = (n+2)/(n-2)$, Δ_g is the Laplace-Beltrami operator in the g-metric, and k is the scalar curvature of (M, g). In the case where M is compact, lots of work has been done and we refer the reader to the monograph by Kazdan [K] for a survey. In the case where M is complete, noncompact, little was known until recently. The first natural case is that $M = \mathbb{R}^n$ and g is the usual metric. In this case equation (1.1) reduces (after a rescaling) to

$$\Delta u + Ku^{n^*} = 0 \tag{1.2}$$

on $\mathbb{R}^n$, where

$$\Delta = \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}.$$

In the case where K decays fast at ∞, we now have a good understanding of the equation (1.2). In fact, in this case we have fairly detailed knowledge about the set of *all* positive entire solutions of the more general equation

$$\Delta u + Ku^p = 0, \tag{1.3}$$

where $p > 1$, in $\mathbb{R}^n$. The first general existence results are due to Ni [N1] in 1982.

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