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SPACE CURVES WHICH ARE THE INTERSECTION OF A CONE WITH ANOTHER SURFACE

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CONTENTS

Introduction. Kneser [11] has proved that every curve $C \subset \mathbb{P}^3$ may be described (set-theoretically) as the intersection of three surfaces. It is conceivable that every (connected) curve C is the intersection of two surfaces; no one has discovered a counterexample. But there is no shortage of candidates—for instance, when the ground field has characteristic zero, we do not know of a single nonsingular curve C which is the intersection of two surfaces and such that $\text{degree}(C) > \text{genus}(C) + 3$. We would like to know exactly which nonsingular curves may be obtained as the intersection of two surfaces.

To simplify matters, one may fix some class of surfaces in $\mathbb{P}^3$, and then ask which smooth curves may be obtained as the (set-theoretic) complete intersection of such a surface with some other, arbitrary surface. For instance, the only smooth curves which may be obtained as the set-theoretic complete intersection of a *smooth* surface with some other surface are the (scheme-theoretic) complete intersection curves.[1] These are easily classified up to degree and genus.

In this article we shall consider the problem of determining which smooth curves may be expressed as the set-theoretic complete intersection of a *cone* with some other surface. Let $S \subset \mathbb{P}^3$ be a cone. Let $C \subset S$ be a smooth curve. We will give necessary and sufficient conditions for there to exist a surface $T \subset \mathbb{P}^3$ such that $C = S \cap T$ as sets. If this happens, we shall say that C is a *set-theoretic complete intersection on S*.

We now give a precise statement of our result. For simplicity, we will suppose that C is not a line. Choose a plane $H \subset \mathbb{P}^3$ which does not contain C or the

[1] This follows from the fact that $\text{Pic}(S)/\text{Pic}(\mathbb{P}^3)$ is torsion-free for any smooth surface $S \subset \mathbb{P}^3$ (see, e.g., [1], 1.8).

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