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A MAXIMUM PRINCIPLE AT INFINITY FOR MINIMAL SURFACES AND APPLICATIONS

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The aim of this paper is to study asymptotic ends of two complete minimal surfaces of finite total curvature. Let us suppose that the ends have no ramification, that is, both surfaces are graphs over the plane orthogonal to the limiting normal. There are two ways to say that one surface is above the other near the end. The most natural is to suppose the limit plane is horizontal and compare the third coordinate of points on the two surfaces having the same horizontal projection. The second viewpoint is to compare the support functions of the two surfaces (see below). The proof of our main theorem will relate the two viewpoints.

Then we will apply this theorem to the "exterior Plateau problem," which is, given $f \in C^\infty(\partial\Omega)$, where $\Omega = \{X = (x_1, x_2) | x_1^2 + x_2^2 \geqslant 1\}$, to prove the existence and unicity of extensions of f to Ω satisfying the minimal surface equation. Bers has already shown there is at most one bounded solution to this problem [O].

We prove unicity of solutions having the same logarithmic growth rate and limiting normal at infinity. We show there are no solutions to the exterior Plateau problem having growth rate greater than one, and the only solution with growth rate one is the catenoid.

Maximum principle at ∞. *Let f_1, f_2 be solutions of the exterior Plateau problem on Ω. Suppose $f_1(X) \geqslant f_2(X)$ for $X \in \Omega$ and there exists a sequence $X_i \in \Omega$ such that $|X_i| \to \infty$ and $|f_1(X_i) - f_2(X_i)| \to 0$. Then $f_1 = f_2$.*

Preliminaries. In this section we derive the Weierstrass representation (g, ω) of a minimal surface in R^3. This is well known to experts in the subject but not to the general public.

A minimal surface in R^3 is a Riemann surface M and a conformal map X: $M \to R^3$ whose coordinate functions are harmonic. The Weierstrass representation is a manner of expressing X in terms of a meromorphic map g on M and a holomorphic one-form ω, with (g, ω) satisfying certain conditions. The map g is the Gauss map of $X(M)$, composed with stereographic projection.

Let D be an open disc in the complex plane and $z = u + \dot{\omega}$ be a local parameter of M; $z \in D$. We write $X(z) = (x_1(z), x_2(z), x_3(z))$. X conformal means $X_u = X_v$ (X_u means partial derivative with respect to u) and $\|X_u\| = \|X_v\|$.

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