Vol. 57, No. 3 DUKE MATHEMATICAL JOURNAL © December 1988

BOUNDARY VALUE PROBLEMS FOR THE SYSTEMS OF ELASTOSTATICS IN LIPSCHITZ DOMAINS

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Introduction. The main purpose of this work is to study the solvability of the Dirichlet problem, and the traction boundary value problem for the Lamé systems of linearized elastostatics on an arbitrary Lipschitz domain Ω in $\mathbb{R}^n$. We obtain existence and uniqueness results, with optimal estimates, assuming that the traction data are in $L^2(\partial\Omega)$, and that the Dirichlet data are in $L^2(\partial\Omega)$, or have first derivatives in $L^2(\partial\Omega)$. We also obtain representation formulas for the solution in terms of layer potentials, thus extending to Lipschitz domains, classical results of Lichtenstein, H. Weyl, Kupradze and others (see Kupradze's book [15]), which were obtained for smooth domains. We are also able to treat the 'slip condition' for the Stokes system of linearized hydrostatics, when the data is in $L^2(\partial\Omega)$, with optimal estimates, thus completing the results in [9]. In recent years, considerable attention has been given to the Dirichlet and Neumann problem for Laplace's equation in a Lipschitz domain, with $L^p(\partial\Omega)$ data, for appropriate p. Thanks to the results in [3], [11], [20], and [4], this is very well understood now. (See for example the introduction to [9] for a detailed description of these developments.)

In this paper, and in its companion paper [9], we initiate the study of boundary value problems on Lipschitz domains for systems of elliptic equations with $L^2(\partial\Omega)$ data. The present work deals mainly with the Lamé systems of linear elastostatics, while [9] deals with the Stokes system of linear hydrostatics.

Our main results are as follows: Let Ω be a bounded Lipschitz domain in $\mathbb{R}^n$, $n \geqslant 3$, with connected boundary. Consider, for $\vec{g} \in L^2(\partial\Omega)$, the Dirichlet problem

$$(\mathrm{D}) \qquad \begin{cases} \mu\Delta\vec{u} + (\lambda+\mu)\nabla(\operatorname{div}\vec{u}) = \vec{0} \text{ in } \Omega \\ \vec{u}|_{\partial\Omega} = \vec{g} \text{ a.e. on } \partial\Omega \text{ in the sense of nontangential convergence} \\ \|\vec{u}^*\|_2 < \infty \end{cases}$$

where $(\ )^*$ denotes the nontangential maximal function (see the body of the paper for the relevant definitions), where $\mu > 0$, and $\lambda > -2\mu/n$ are constants (Lamé moduli). Then, there exists a unique $\vec{u}$ satisfying (D) (Theorem 3.6). Moreover, if in addition $\vec{g} \in L_1^2(\partial\Omega)$, i.e., it has first derivatives in $L^2(\partial\Omega)$, the solution

Received December 15, 1987. All authors supported in part by N.S.F. Second author supported in part by the J. S. Guggenheim Memorial Foundation.