Vol. 57, No. 3 DUKE MATHEMATICAL JOURNAL © December 1988

THE DIRICHLET PROBLEM FOR THE STOKES SYSTEM ON LIPSCHITZ DOMAINS

E. B. FABES, C. E. KENIG, AND G. C. VERCHOTA

Introduction. The main purpose of this work is to study the solvability of the Dirichlet problem for the Stokes system of linearized hydrostatics on an arbitrary Lipschitz domain Ω in $\mathbb{R}^n$. We obtain existence and uniqueness results, with optimal estimates, assuming that the boundary values are in $L^2(\partial\Omega)$, or have first derivatives in $L^2(\partial\Omega)$. We also obtain representation formulas for the solution in terms of layer potentials, thus extending to Lipschitz domains, classical results of Odqvist and Lichtenstein, which were obtained for smooth domains (See Chapter 3 of Ladyzhenskaya's book [17]).

In recent years, considerable attention has been given to the Dirichlet and Neumann problem for Laplace's equation in a Lipschitz domain, with $L^p(\partial\Omega)$ data, or data with one derivative in $L^p(\partial\Omega)$. This started with the work of B. Dahlberg ([4], [5]) on the Dirichlet problem, with data in $L^p(\partial\Omega)$ and optimal estimates. Dahlberg's techniques relied on positivity, and thus were not applicable to the Neumann problem or to systems of equations. Shortly afterward, E. Fabes, M. Jodeit, Jr. and N. Riviere ([11]) were able to utilize A. P. Calderón's theorem [1] on the boundedness of the Cauchy integral on C^1 curves, to extend the classical method of layer potentials to the case of C^1 domains. They were thus able to resolve the Dirichlet problem for Laplace's equation with data in $L^p(\partial\Omega)$, and with one derivative in $L^p(\partial\Omega)$, and the Neumann problem with data in $L^p(\partial\Omega)$, with optimal estimates, on C^1 domains. They relied on the Fredholm theory, exploiting the compactness of the appropriate layer potentials in the C^1 case.

In [12], D. Jerison and C. Kenig were able to give a new proof of Dahlberg's results, using an integral identity that goes back to Rellich ([19]). The method however still relied on positivity. Soon after that, D. Jerison and C. Kenig ([13]) were also able to give optimal estimates for the Dirichlet problem when the data has one derivative in $L^2(\partial\Omega)$, and for the Neumann problem with $L^2(\partial\Omega)$ data. To do so, they combined the Rellich formulas with Dahlberg's result. This of course still restricted to applicability of the method to a single equation.

In 1981, R. Coifman, A. McIntosh and Y. Meyer [3], established the boundedness of the Cauchy integral on any Lipschitz domain, opening the door to the applicability of the layer potential method to Lipschitz domains. This method is very flexible, and does not in principle differentiate between a single equation or

Received December 15, 1987. All authors supported in part by N.S.F. Second author supported in part by the J. S. Guggenheim Memorial Foundation.