Vol. 57, No. 1 DUKE MATHEMATICAL JOURNAL © August 1988

LOCAL REGULARITY OF *CR* HOMEOMORPHISMS

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1. Introduction. In this note, we shall prove the following result:

THEOREM 1. *Suppose* $f: M_1 \to M_2$ *is a CR homeomorphism between open connected* C^∞ *smooth pseudoconvex hypersurfaces in* $\mathbb{C}^n$ $(n > 1)$. *If* M_1 *and* M_2 *are of finite type in the sense of D'Angelo* [8], *then f must be a local* C^∞ *diffeomorphism.*

Here, we call a mapping *CR* homeomorphic if it is a continuous *CR* map with a continuous inverse which is also *CR*. In the course of the proof of this theorem, we shall see that in fact f locally extends to be a holomorphic mapping on the pseudoconvex side of M_1 in $\mathbb{C}^n$ and the extension of f is C^∞ up to M_1.

This theorem extends and improves some known local extendibility results for holomorphic mappings (see [11, 10, 4]). The proof of Theorem 1 will use the techniques developed in [4] for studying the boundary behavior of proper holomorphic mappings between smooth domains in $\mathbb{C}^n$. In order to apply these techniques, we must first show that if $f(z_0) = w_0$, then f extends to the pseudoconvex side of M_1 near z_0 in such a way that it maps a small domain D_1 biholomorphically onto a domain D_2, where D_1 is a pseudoconvex domain whose boundary contains a neighborhood of z_0 in M_1, and D_2 is a domain whose boundary contains a neighborhood of w_0 in M_2. This will be done in §2. In §3, it will be shown that, under these circumstances, the extension of f is C^∞ smooth up to M_1.

Theorem 1 has applications to the problem of holomorphic extension of *CR* maps. The following theorem is a direct consequence of Theorem 1 and results proved by Baouendi, Jacobowitz, and Treves [1]:

THEOREM 2. *If* $f: M_1 \to M_2$ *is a CR homeomorphism between pseudoconvex real-analytic hypersurfaces in* $\mathbb{C}^n$ *which are of finite type in the sense of D'Angelo, then f extends holomorphically to an open set in* $\mathbb{C}^n$ *which contains* M_1.

Acknowledgement. I would like to thank M. S. Baouendi for suggesting this problem to me and for pointing out the importance of Theorem 3, below. I would also like to thank J.-P. Rosay for a helpful discussion in which he told me the basic idea for the proof of Theorem 3.

2. Extension to the pseudoconvex side. In this section, we will prove some results which lead up to a proof of the existence of the local domains D_1 and D_2 mentioned in §1. There are two popular notions of finite type for a hypersurface

Received June 24, 1987. Research partially supported by Sloan fellowship and NSF grant DMS 8420754.