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A LOWER BOUND FOR THE VOLUME OF HYPERBOLIC 3-ORBIFOLDS

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1. Introduction. In this paper, we prove

THEOREM. *All complete orientable hyperbolic* 3*-orbifolds have volume at least* 0.0000013. *All cusped complete orientable hyperbolic* 3*-orbifolds have volume at least* $\sqrt{3}/24 \approx 0.07217$.

Before discussing the proof of this theorem, it is important to clarify the relationship between the papers [M1], [M2], [M3], and the present paper. The results of [M2] and [M3] are better than the present results, but they depend crucially on the results of [M1] and the present paper. Roughly speaking, [M1] and the present paper show how to construct reasonable-sized objects in arbitrary hyperbolic 3-manifolds and 3-orbifolds, thereby producing a lower bound for the volume of hyperbolic 3-manifolds and 3-orbifolds. The question remains as to how much volume lies outside of the constructed objects. This question is attacked in [M2] and [M3] by means of sphere packing, thereby improving the lower bounds.

The results of [M1] and the present paper were obtained before the results of [M2] and [M3]. This makes sense because the results of [M2] and [M3] are enhancements of the earlier results. The confusing point is that the papers [M2] and [M3] appeared before the "earlier" papers. Also, it should be noted that although the present paper is organized around the proof of the above theorem, the constructions used to prove the theorem are of independent interest. In particular, the construction of "solid tubes" around "short geodesics" in hyperbolic 3-orbifolds is an important ingredient in understanding the structure of hyperbolic 3-orbifolds—roughly equivalent to the Margulis constant.

It is assumed that the reader is familiar with the background material and notation outlined in the first three paragraphs of [M2] and [M3]. It will not be repeated here. The only significant change from those paragraphs is that here we will work with PSL(2, **C**) rather than PGL(2, **C**). The transition is easy enough in our context and enables us to make use of Jørgensen's trace inequality.

The proof of the above theorem is a generalization of the technique used in [M1]. We outline that technique here. The (complex) lengths of geodesics in a hyperbolic 3-manifold $M = H^3/\Gamma$ correspond to the traces of appropriate matrices in $\Gamma \subseteq \mathrm{PSL}(2, \mathbf{C})$. (The complex length of a geodesic is a complex number whose real part is the standard [real] length of the geodesic and whose imaginary part is the holonomy angle incurred in travelling once along the

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