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SPECTRA OF MANIFOLDS LESS A SMALL DOMAIN

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Let M be a compact, connected C^∞ Riemannian manifold and Δ the Laplace–Beltrami operator, associated to the Riemannian metric, acting on functions on M. We also let

$$\{0 = \lambda_0 < \lambda_1 \leqq \lambda_2 \leqq \cdots \uparrow +\infty\}$$

denote the spectrum of Δ, with each eigenvalue repeated according to its multiplicity.

Let M^* be a compact submanifold of M, $\varepsilon > 0$, and B_ε the tubular neighborhood of M^* of radius ε. To B_ε we associate the restriction Δ_ε, of Δ, to those functions on M vanishing identically in B_ε. Then Δ_ε has spectrum

$$\{0 < \lambda_{1;\varepsilon} \leqq \lambda_{2;\varepsilon} \leqq \lambda_{3;\varepsilon} \leqq \cdots \uparrow +\infty\},$$

with each eigenvalue repeated according to its multiplicity. That is, $\lambda_{j;\varepsilon}$ is the jth Dirichlet eigenvalue of

$$\Omega_\varepsilon \equiv: M \setminus \overline{B_\varepsilon}.$$

In [4] it was proved that if

$$\ell \equiv: \dim M - \dim M^* \geqq 2,$$

then

$$\lim_{\varepsilon \downarrow 0} \lambda_{j;\varepsilon} = \lambda_{j-1} \tag{1}$$

for all $j = 1, 2, \ldots$. In this note we sharpen (1) to obtain the first correction term of the asymptotic expansion of $\lambda_{j;\varepsilon}$ with respect to ε, viz.,

THEOREM 1. *Let λ_{j-1} have multiplicity equal to 1, and ϕ_{j-1} be an $L^2(M)$-normalized eigenfunction of λ_{j-1}. Then, for $\ell > 2$ and $k \equiv: \dim M^*$, we have*

$$\lambda_{j;\varepsilon} \sim \lambda_{j-1} + (\ell - 2)\mathbf{c}_{\ell-1}\varepsilon^{\ell-2} \int_{M^*} \phi_{j-1}^2(y)\, dV_k(y) \tag{2}$$

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