Vol. 56, No. 2 DUKE MATHEMATICAL JOURNAL © April 1988

COMPLETE CONFORMAL METRICS WITH NEGATIVE SCALAR CURVATURE IN COMPACT RIEMANNIAN MANIFOLDS

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In [3], Loewner and Nirenberg considered the following question: Given a smooth submanifold Γ in the sphere S^n, when is there a complete, conformally Euclidean metric $\hat{g}$ on $S^n \setminus \Gamma$ with constant negative scalar curvature? The answer is that such a metric exists if and only if $d = \dim \Gamma > (n-2)/2$. (Cf. [1], [3], and [5].) In this paper we observe that the same conclusions hold if S^n is replaced by *any* compact Riemannian manifold without boundary:

THEOREM. *Suppose (M, g) is a compact Riemannian manifold of dimension $n \geqslant 3$ and let Γ be a closed smooth submanifold of dimension d. Then there is a complete conformal metric $\hat{g}$ on $\hat{M} = M \setminus \Gamma$ with constant negative scalar curvature if and only if $d > (n-2)/2$.*

Our proof of the existence of $\hat{g}$ generalizes that of [3] when the first eigenvalue λ_0 of the "conformal Laplacian" on $\hat{M}$ is nonnegative, but for $\lambda_0 < 0$ we must invoke a result from [2]; to prove the necessity of the condition on d, we must abandon the explicit solutions used in [3] in favor of an analysis related to [1].

As a special case we may take $d = n - 1$ so that $(\hat{M}, g)$ is a compact Riemannian manifold with boundary (not necessarily connected). In fact, if we start with a compact Riemannian manifold $(\hat{M}, g)$ with boundary $\Gamma = \partial \hat{M}$, we may embed it in a manifold without boundary, extend g, and apply the above to obtain:

COROLLARY. *Any compact Riemannian manifold with boundary admits a complete conformal metric with constant negative scalar curvature.*

Note. All manifolds, submanifolds, and metrics in this paper are assumed to be smooth, i.e., C^∞. In particular, the metric on a manifold with boundary is assumed to be smoothly extendible to a neighborhood of the boundary.

1. Proof of "if." We want to find a positive solution of

$$\Delta u - u^{(n+2)/(n-2)} = \frac{n-2}{4(n-1)} S u \quad \text{in} \quad \hat{M} = M \setminus \Gamma$$

$$u \to +\infty \quad \text{as} \quad x \to \Gamma,$$

where S denotes the scalar curvature. We consider cases depending on the sign of

Received May 11, 1987.