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VANISHING CYCLES, RAMIFICATION OF VALUATIONS, AND CLASS FIELD THEORY

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Introduction. The purpose of this paper is to give a generalization of a theorem of Deligne on vanishing cycles in relative dimension one

$$\dim\left(\psi_x^1(u_!\mathscr{F})\right) = \varphi(\eta) - \varphi(s)$$

([9], Theorem 5.1.1; the notations will be reviewed below). In this theorem, the sheaf $\mathscr{F}$ is assumed to be unramified at the generic point of the special fiber. We consider in this paper the ramified case, relating it to a ramification theory of valuation rings of rank two, and to the two dimensional local class field theory ([6], [12], [20]).

To be precise, let O_k be an excellent henselian discrete valuation ring with field of fractions k and with algebraically closed residue field F, and let A be the henselization $O_k\{T\}$ of the local ring of the polynomial ring $O_k[T]$ at the maximal ideal $\mathrm{Ker}(O_k[T] \to F;\ T \mapsto 0)$. Let $u\colon U \hookrightarrow \mathrm{Spec}(A \otimes_{O_k} k)$ be a non-empty open subscheme, Λ a field of positive characteristic such that $\mathrm{char}(\Lambda) \neq \mathrm{char}(F)$, and let $\mathscr{F}$ be a Λ-module on the étale site $U_{\text{ét}}$ which is locally constant of finite rank. We denote the algebraic closure of k by $\bar{k}$, the closed point of $\mathrm{Spec}(A)$ by x, the generic point of $\mathrm{Spec}(A \otimes_{O_k} F)$ by $\mathfrak{p}$, the residue field of $\mathfrak{p}$ by $\kappa(\mathfrak{p})$, and the local ring of A at $\mathfrak{p}$ by $A_{\mathfrak{p}}$. Then, the space of vanishing cycles

$$\psi_x^q(u_!\mathscr{F}) = H^q_{\text{ét}}\left(\mathrm{Spec}\left(A \otimes_{O_k} \bar{k}\right), u_!\mathscr{F}\right) \qquad (q \geqq 0)$$

is zero for $q \geqq 2$, and is computed easily for $q = 0$. The theorem of Deligne gives the dimension of $\psi_x^1(u_!\mathscr{F})$ assuming that $\mathscr{F}$ is unramified with respect to the

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