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HODGE NUMBERS OF LINKED SURFACES IN $\mathbb{P}^4$

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Introduction. In [O1, O2, O3], Okonek has classified smooth surfaces in $\mathbb{P}^4$ of degree $\leqslant 8$, and found that each belongs to one of three liaison classes. His method requires the computation of several invariants of such surfaces.

That result suggests the utility of using liaison to construct and identify surfaces of low degree in $\mathbb{P}^4$. The standard technique for identifying projective surfaces is the adjunction mapping [S]; however the invariant K^2 is needed to use this method. K^2 can often be determined by ad hoc techniques; what is new here is a systematic method for computing K^2 of a linked surface. This is accomplished by computing its Hodge structure, which in turn is computed via the Clemens-Schmid sequence. The surprise was that the scope of these methods is broader than was originally anticipated by the author—not only did this lead to the construction of a previously undiscovered class of surfaces (degree 9 K3's with 5 points blown up), but also it applied to answer a question of Okonek [O3] by showing that the ideal of a certain rational octic is not generated by quartics.

The paper is organized as follows. In Section 1, we give the main construction. In Section 2, we analyze the degeneration of Section 1 via the Clemens-Schmid sequence to yield the main computational result, Corollary 2.4. Section 3 is devoted to applications. We compute invariants of all surfaces linked to a plane or to a Veronese surface, and classify these surfaces, in particular constructing the degree 9 K3 surface with 5 points blown up mentioned above. We also answer the question of Okonek.

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1. The set-up. Let $S, S' \subset \mathbb{P}^4_{\mathbf{C}}$ be smooth surfaces of degrees d, d' respectively.

Definition. S, S' are *nicely linked* if:
(a) $S \cup S'$ is a complete intersection $G \cap F$ of hypersurfaces
(b) $S \cap S'$ is a smooth curve C, and
(c) G may be chosen to be smooth away from C, with finitely many nodes on C.

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