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# GROTHENDIECK GROUPS OF POLYNOMIAL AND LAURENT POLYNOMIAL RINGS

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For any noetherian scheme $T$, recall that

$$NK_0(T) = \operatorname{coker}(K_0(T) \to K_0(T \times \mathbf{A}^1))$$

$$K_{-1}(T) = \operatorname{coker}(K_0(T \times_{\mathbf{Z}} \operatorname{Spec} \mathbf{Z}[t]) \oplus K_0(T \times_{\mathbf{Z}} \operatorname{Spec} \mathbf{Z}[t^{-1}]) \to K_0(T \times_{\mathbf{Z}} \operatorname{Spec} \mathbf{Z}[t, t^{-1}])).$$

Here $K_0$ denotes the Grothendieck group of vector bundles (locally free sheaves of finite rank). It is well known that if $T$ is regular, then $NK_0(T) = K_{-1}(T) = 0$. If $T$ is a nonnormal scheme, then various simple examples exist with $NK_0(T) \neq 0$ or $K_{-1}(T) \neq 0$; for example $NK_0(T) \neq 0$ for $T = \operatorname{Spec}(k[t^2, t^3])$ while $K_{-1}(T) \neq 0$ for $T = \operatorname{Spec}(k[t^2 - 1, t^3 - t])$. However it is more difficult to construct examples of normal varieties with $NK_0 \neq 0$.

Murthy and Pedrini [MP] showed that $NK_0 = 0$ for certain surfaces with isolated rational singularities. In [W1] Weibel gave the first example of a normal ring in positive characteristic with $NK_0 \neq 0$, based on Example 6 of the appendix to Nagata's book *Local Rings* [N]. In the same paper, Weibel discusses examples of Swan of normal affine hypersurfaces of dimension $\geq 3$ with $NK_0 \neq 0$, where the equation of the hypersurface is of the form $x_0 x_1 = f(x_2, \dots, x_n)$ and $f(x_2, \dots, x_n) = 0$ in $\mathbf{A}^{n-2}$ is nonnormal.[1]

One way to construct examples with $NK_0 \neq 0$ is to use a remark of Swan and Weibel that for a graded ring $A = \bigoplus_{n \geq 0} A_n$, $K_0(A) \cong K_0(A[t])$ implies that $K_0(A) \cong K_0(A_0)$. Thus if $A_0 = k$ is a field, and $K_0(A) \not\cong \mathbf{Z}$, then $NK_0(A) \neq 0$. Using this, Bloch and Murthy (unpublished, but see [S1]) showed that $NK_0(A) \neq 0$ for $A = \mathbf{C}[x, y, z]/(z^2 + x^3 + y^7)$.

In [S1] the author used relative $K$-theory to give numerous examples of affine cones over projectively normal curves over $\mathbf{C}$ with $K_0 \neq \mathbf{Z}$; by the Swan-Weibel remark these cones have $NK_0 \neq 0$. The examples are the cones over curves $C \subset \mathbf{P}^n$ with $H^1(C, \mathcal{O}_C(1)) \neq 0$ i.e., curves embedded by a special linear system. In characteristic $p > 0$, the author showed that $K_0 = \mathbf{Z}$ for any (positively) graded 2-dimensional affine domain over an algebraically closed field, so the

[1] See also [R].

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