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A SCATTERING THEORY FOR TIME-DEPENDENT LONG-RANGE POTENTIALS

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§1. Introduction. Theorems. In this paper, we study the scattering theory for Schrödinger equations with *time-dependent* potentials which are possibly *long-range* in space direction. We prove the existence of the modified wave operators and characterize their ranges in terms of the space-time behavior of the wave packet. If the potential is periodically dependent or independent of time, the asymptotic completeness of the modified wave operators is obtained by applying this characterization theorem to Ruelle [24], Amrein–Georgescu [2] or Veselič [27] criteria for the continuous spectrum. We also show that our result can be applied to some equations of semiclassical theory of radiation called AC-Stark effect by Howland [9]. Basically we follow Enss method [6], however, we bring a new ingredient into it; the machinery of Fourier integral operators, which clarifies and simplifies the theory and extends the scope of the application of Enss method to include the potentials which will be considered here.

We consider the Schrödinger equation

$$i\partial u/\partial t = H(t)u \equiv (H_0 + V(t))u, \qquad H_0 = -\frac{1}{2}\sum_{j=1}^{N} \partial^2/\partial x_j^2 \equiv -\frac{1}{2}\Delta \quad (1.1)$$

in the Hilbert space $\mathfrak{H} = L^2(\mathrm{R}^N)$ with the time-dependent perturbation $V(t) = V^L(t,x) + V^S(t)$. We assume the following conditions (L), (S), and (G). Roughly speaking, they require that (L) the long-range part $V^L(t,x)$ is a smooth function of x; (S) the short-range part $V^S(t)$ may be velocity dependent or singular but is $(H_0+1)^{1/2-\epsilon}$-bounded; and (G) the equation (1.1) generates a unitary propagator. We state them precisely.

Assumption (L). (i) $V^L(t,x)$ is a real-valued function of $(t,x) \in \mathrm{R}^{N+1}$.

(ii) For each $t \in \mathrm{R}^1$, $V^L(t,\cdot) \in C^\infty(\mathrm{R}^N)$.

(iii) For any multi-index α, $\partial_x^\alpha V^L(t,x) \in C(\mathrm{R}^{N+1})$.

(iv) There exists a constant $0 < \epsilon \leq 1$ such that for any multi-index α,

$$\left|\partial_x^\alpha V^L(t,x)\right| \leq C_\alpha (1+|x|)^{-|\alpha|-\epsilon}, \qquad (t,x) \in \mathrm{R}^{N+1} \quad (1.2)$$

Assumption (S). (i) For each $t \in \mathrm{R}^1$, $V^S(t)$ is a symmetric operator in $\mathfrak{H}$ with domain $\mathfrak{D}(V^S(t)) \supset \mathfrak{D}(H_0) = H^2(\mathrm{R}^N)$ (Sobolev space of order two).

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