

STRONG RIGIDITY OF COMPACT QUOTIENTS OF EXCEPTIONAL BOUNDED SYMMETRIC DOMAINS

YUM-TONG SIU

In [5] we prove the following theorem in the case where the bounded symmetric domain is one of the four classical types.

MAIN THEOREM. *Suppose M is a compact quotient of an irreducible bounded symmetric domain of complex dimension at least two. If N is a compact Kähler manifold which is of the same homotopy type as M , then N is either biholomorphic or antibiholomorphic to M .*

In this paper we complete the proof of the Main Theorem by doing the cases of the two exceptional bounded symmetric domains. The proof for the cases of the four types of classical bounded symmetric domains consists of using harmonic maps, deriving a Bochner type formula, and a tedious part involving complicated linear algebra manipulations to verify that the curvature conditions from the Bochner type formula are satisfied by the classical bounded symmetric domains. In this paper we reformulate the curvature conditions so that they can be verified directly from the root system without using any explicit expression of the curvature tensor in terms of the coordinates of a realization of the bounded symmetric domain (see §2 and §3). We prove the strong rigidity for the two exceptional cases by using this reformulation (see §4 and §5).

This reformulation of the curvature condition, when applied to the cases of the first three classical types, can yield a proof of the strong rigidity for these cases which is simpler than that given in [5] (see the Remark in §4). (The strong rigidity proof given in [5] for the fourth classical type is already very simple.)

We now know an alternative method of deriving the Bochner type formula for harmonic maps between compact Kähler manifolds. This alternative method does not use the special trick of considering the wedge product instead of the square norm of the differential of a harmonic map. It is more natural and we give this alternative derivation in this paper (see §1).

At the end of this paper (§6) we discuss a conjecture concerning the analyticity of harmonic maps with sufficiently high rank into compact quotients of bounded symmetric domains.

We would like to thank the referee for suggesting the present simple proof of Proposition (4.1) which replaces our original lengthy case-by-case verification and for suggesting the alternative proof of Proposition (5.1). In the meantime,

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