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EXAMPLES OF SINGULAR PARABOLIC MEASURES AND SINGULAR TRANSITION PROBABILITY DENSITIES

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We consider strongly parabolic operators

$$L = \sum_{i,j=1}^{n} a_{ij}(t,x)D^2_{x_i x_j} - D_t,$$

i.e., we assume there exists $\lambda > 0$ such that for each $t > 0$, $x \in R^n$, $\xi \in R^n$, $a_{ij}(t,x) = a_{ji}(t,x)$ and

$$\lambda|\xi|^2 \leqslant \sum_{i,j=1}^{n} a_{ij}(t,x)\xi_i\xi_j \leqslant \frac{1}{\lambda}|\xi|^2.$$

If in addition, the coefficients are uniformly continuous then the classical Cauchy or initial value problem,

$$Lu = 0, t > 0$$
$$u(0,x) = g(x)$$

is uniquely solvable for continuous g with compact support in R^n. The solution $u(t,x)$, satisfies the condition

$$\sum_{|\alpha| \leqslant 2} \int_a^b \int_{R^n} \left[|D_x^\alpha u|^p + |D_t u|^p \right] dx\, dt < \infty \tag{i}$$

for each $0 < a < b < \infty$ and each $1 < p < \infty$. In particular u is continuous in $R^{n+1}_+ = (0,\infty) \times R^n$, uniformly continuous in $[a,b] \times R^n$, for each $0 < a < b < \infty$ and in addition satisfies

$$\lim_{t \to 0^+} \sup_x |u(t,x) - g(x)| = 0. \tag{ii}$$

The maximum principle implies that the mapping

$$g \to u(t,x)$$

is a positive continuous linear functional on $C_0(R^n)$, the space of continuous functions with compact support. Hence there exists a locally finite (actually finite

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