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ARITHMETIC OF DIFFERENTIAL OPERATORS ON SYMMETRIC DOMAINS

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Introduction. Throughout the paper, we denote by $\mathcal{H}$ the Siegel upper half space of degree m, which consists of all complex symmetric matrices of size m with positive definite imaginary part. If $m = 1$, our operators have the forms

$$D_r^k = D_{r+2k-2} \cdots D_{r+2}D_r, \qquad D_r = (z - \bar{z})^{-1}r + \partial/\partial z \qquad (r \in \mathbf{Z}, z \in \mathcal{H}).$$

Operator D_r^k sends a (holomorphic or nonholomorphic) modular form of weight r to a form of weight $r + 2k$. Moreover, if f and g are elliptic modular forms of weight r and $r + 2k$ respectively, and if they have algebraic Fourier coefficients, then $\pi^{-k}g^{-1}D_r^kf$ takes an algebraic value at every imaginary quadratic point, as proved in [8]. Now the purpose of the present paper is to define some operators with similar properties on $\mathcal{H}$ with $m \geqslant 1$ and also on other domains, and to study the arithmetic nature of the values of certain nonholomorphic Eisenstein series at CM-points on $\mathcal{H}$ by means of the operators. We have already shown in our previous paper [14] how these ends are attained in the case of the orthogonal group $SO(m, 2)$. Although the same ideas are applicable to the symplectic and other groups, there are several interesting aspects of the theory of such operators which were not dealt with in [14]. Therefore we treat here the symplectic case anew, considering in particular the features not covered by what we did in [14].

The definition of a generalization of D_r in the case $m \geqslant 1$ is relatively simple. In fact, fixing a rational representation

$$\rho : GL_m(\mathbf{C}) \to GL(V)$$

with a finite-dimensional complex vector space V, we take $\rho(cz + d)$ as the factor of automorphy with the standard meaning of $cz + d$. Then, for a V-valued function f on $\mathcal{H}$, we put

$$D_\rho f = \rho(z - \bar{z})^{-1}D(\rho(z - \bar{z})f),$$

where $D = ((1/2)(1 + \delta_{ij})\partial/\partial z_{ij})$; we understand that $D_\rho f$ has values in $\mathrm{Hom}(\mathbb{S}, V)$, where $\mathbb{S}$ is the global complex tangent space of $\mathcal{H}$. This operator has a certain property of commutativity with the action of the symplectic group. It is important, however, to know the nature of iterated operators of type $D_\omega \cdots D_\sigma D_\rho$, which are generalizations of the above D_r^k and considerably more

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