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SPECTRAL ASYMPTOTICS FOR THE $\bar{\partial}$-NEUMANN PROBLEM

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0. Introduction. With the rather extensive study of the $\bar{\partial}$-Neumann problem (see for instance Hörmander [5], Folland–Kohn [3], and the references there) it may be of some interest to give an asymptotic formula for the eigenvalues of this self adjoint nonelliptic boundary value problem.

When considering the problem on $\Omega \subset \mathbf{C}^n$ we have (in a sense which is made precise in [9]):

$$N(\lambda) \sim c_i \lambda^n + B(\lambda) \tag{0.1}$$

where $N(\lambda)$ denotes the number of eigenvalues less or equal to λ, c_i is the "usual interior constant" for elliptic problems, and $B(\lambda)$ measures the contribution of the boundary. For elliptic boundary value problems it is well known that $B(\lambda)$ is negligible in front of the interior term, while for some degenerate problems the opposite phenomenon is occurring (see for instance [10], [12] . . .). Here, using min-max arguments, one can show, when $\partial\Omega$ is a C^∞ manifold, that $B(\lambda)$ is equivalent to the counting function $N_b(\lambda)$ of the eigenvalues of a pseudodifferential operator on the boundary $\partial\Omega$. When this operator is subelliptic with loss of one derivative (and in our case, that means that condition $Z(q)$ is satisfied), one could make use of the result of Menikoff–Sjöstrand [8] (suitably extended to systems) and obtain that

$$N_b(\lambda) \sim c_b \lambda^n. \tag{0.2}$$

So it appears that for the $\bar{\partial}$-Neumann problem the boundary term $B(\lambda)$ has the same order of growth as the interior term $c_i\lambda^n$, and one can expect a formula of the kind

$$N(\lambda) \sim (c_i + c_b)\lambda^n. \tag{0.3}$$

However, using the ideas of [11], we will give in this paper a self contained and independent proof of (0.3), the main interest of which being that it does not require $\partial\Omega$ to be very smooth (C^2 will be sufficient). Also we will localize (0.3) and show that the spectral function satisfies (see section 2 for a precise statement)

$$\operatorname{tr} e(\lambda; z, z) \sim c_0 \lambda^n + \lambda^{n+1} \int_0^\infty e^{-2\lambda\tau\, d(z)} 2\tau c(z,\tau)\, d\tau \tag{0.4}$$

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