Vol. 48, No. 4 DUKE MATHEMATICAL JOURNAL © December 1981

L^p ESTIMATES FOR RADON TRANSFORMS IN EUCLIDEAN AND NON-EUCLIDEAN SPACES

ROBERT S. STRICHARTZ

§1. **Introduction.** The Radon transform in Euclidean spaces assigns to functions on R^n their integrals over all affine hyperplanes. A natural generalization, called the k-plane transform or the X-ray transform, involves integrals over all affine k-dimensional subspaces, where k is fixed, $1 \leqslant k \leqslant n-1$. These operators have been widely studied, and the reader is referred to Helgason [6] and Smith, Solmon and Wagner [14] for expositions of their basic properties and significance. Many generalizations of the Radon transform to spaces other than R^n have been considered; in particular, Helgason [4], [6], considers a Radon transform on the classical non-Euclidean spaces of constant curvature where the hyperplanes are replaced by the totally geodesic submanifolds.

The purpose of this paper is to study the L^p mapping properties of these operators. There are two reasons why we believe this study is important. The first is that we would like to know how the size of a function influences the size and smoothness of its Radon transform, and the L^p spaces furnish a fairly refined measurement of size. The second is that the Radon transform is a natural test case to explore the powers and limitations of our technical understanding of various aspects of harmonic analysis on Euclidean and non-Euclidean spaces.

The Euclidean case, discussed in Section 2, is relatively simple. If Π denotes a fixed k-dimensional linear subspace of R^n and y denotes a variable in $\Pi^{\perp}$, let $L_{\Pi}f(y) = \int_{\Pi} f(y+x)\,dx$. If $f \in L^p(\mathsf{R}^n)$ for $1 < p \leqslant 2$ we will show that $L_{\Pi}f$ has k/p' derivatives in the y-variable in L^p. This result is well known for $p = 2$, and we obtain the general result using an interpolation technique of Fefferman and Stein [2]. Our result is a refinement of a theorem of Solmon [16] who used a less precise interpolation argument. We also obtain some results for $p > 2$ which do not involve smoothness, and these seem to be new. A little thought shows that the only reason we can expect any smoothness at all is that parallel k-planes remain close together; conversely, we can never expect smoothness in the Π-variable because nonparallel k-planes diverge. By the same token we can expect no smoothness in the hyperbolic non-Euclidean case because there are no parallel geodesics.

In section 3 we discuss the Radon transform in hyperbolic non-Euclidean space, restricting attention to the hypersurface case, $k = n-1$. Here the natural tool to use is harmonic analysis, since both the domain and range are homogeneous spaces of the Lorentz group $SO_e(n,1)$ for which the harmonic

Received May 4, 1981. Research supported in part by NSF grant MCS-8002771.