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CR EXTENDABILITY NEAR A POINT WHERE THE FIRST LEVIFORM VANISHES

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1. Introduction. Let M be a smooth generic $2n-d$ real dimensional submanifold of $\mathbf{C}^n$. Let $T(M)$ be the real tangent bundle of M and $T^{\mathbf{C}}(M)$ be its complexification. Let $H(M) \subset T(M)$ be the holomorphic tangent bundle of M and $Y(M) \subset T(M)$ be the totally real subbundle of $T(M)$ $(T(M) = H(M) \oplus Y(M))$. The fiber $H_p(M)$ is the largest J-invariant subspace of $T_p(M)$ ($J :=$ the complex structure map on $\mathbf{R}^{2n}$). If $N_p(M)$ is the orthogonal compliment of $T_p(M)$ in $\mathbf{R}^{2n}$ then $J : Y_p(M) \to N_p(M)$ is an isometry. M is generic means that $\dim_{\mathbf{R}} H_p(M) = 2n - 2d$ (minimal) and $\dim_{\mathbf{R}} Y_p(M) = d$ (maximal) for each $p \in M$.

We let $H^{\mathbf{C}}(M) = H^{1,0}(M) \oplus H^{0,1}(M)$ be the complexification of $H(M)$. The first leviform $\mathfrak{L}_p^1 : H_p^{1,0}(M) \to N_p(M)$ is defined by

$$\mathfrak{L}_p^1(X_p) = \frac{1}{2i} J\left\{\pi_p^1[X, \bar{X}]_p\right\}$$

where $\pi_p^1 : T_p^{\mathbf{C}}(M) \to T_p^{\mathbf{C}}(M)/H_p^{\mathbf{C}}(M)$ is the projection map and $X \in H^{1,0}(M)$ is any smooth vectorfield extension of X_p.

If M is a real hypersurface, then up to a scalar factor $\mathfrak{L}_p^1$ can be identified with the restriction to $H_p^{1,0}(M)$ of the complex hessian matrix of a local defining function for M. In this case, Hans Lewy first showed that if $\mathfrak{L}_p^1$ has eigenvalues of opposite sign (equivalently, the image of $\mathfrak{L}_p^1$ is all of $N_p(M) \simeq \mathbf{R}$) then M is locally CR extendible near p, i.e., each continuous CR function (a solution to the homogeneous tangential Cauchy Riemann equations) near p extends to a holomorphic function defined on an open neighborhood of p in $\mathbf{C}^n$; and furthermore, the set to which the CR function is extended as a holomorphic function depends only on the subset of M on which the CR function is defined.

In [BP], the above theorem of Hans Lewy was generalized to submanifolds of $\mathbf{C}^n$ with higher codimension. There it was shown that if $\Gamma_p :=$ the convex hull of the image of $\mathfrak{L}_p^1$ is all of $N_p(M)$, then M is locally CR extendible near p. Note that if M is a real hypersurface, then the image of $\mathfrak{L}_p^1$ is either a point ($\mathfrak{L}_p^1 \equiv 0$) or a ray (one nonzero eigenvalue) or all of $N_p(M) \simeq \mathbf{R}$ (eigenvalues of opposite sign). Thus, in this case the image of $\mathfrak{L}_p^1$ is always convex.

As far as the author knows, very little is known about CR extendability near a

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