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# THE UNITARY DUAL OF Sp$(n, 1)$, $n \geqslant 2$

M. W. BALDONI SILVA

**0. Introduction.** This paper contains the description of the unitary dual of the group $G = \mathrm{Sp}(n, 1)$, $n \geqslant 2$.

To be more precise we need some notation and we refer to §1 for details.

Let $P = MAN$ be a minimal parabolic subgroup.

If $\xi \in \hat{M}$ and $\nu \in \mathfrak{a}_{\mathbb{C}}^*$ ($\mathfrak{a}$ the Lie algebra of $A$), let $\pi_{\xi,\nu}$ be the principal series representation defined by $\pi_{\xi,\nu} = \mathrm{ind}_P^G(\xi \otimes e^{\nu} \otimes 1)$, where the induction is normalized.

If $\mathrm{Re}\,\nu$ is strictly in the positive Weyl chamber determined by the choice of $N$, denote by $J_{\xi,\nu}$ the Langlands quotient. It is well known that the problem of classifying the unitary irreducible representation of such $G$ amounts to describe the Langlands quotients with parameter on $\mathfrak{a}$ real and bounded by $\rho$ ($\rho$ as in §1), which are unitary (in the algebraic sense). On the other hand this is equivalent to decide whether a particular intertwining operating is semidefinite. This can be very hard to determine directly, since in general there is no formula for such operator explicit enough to allow a straightforward computation.

Fix $\xi \in \hat{M}$ and let us call $0 \leqslant \nu_0 < \cdots < \nu_n \leqslant \rho$ the possible values of $\nu$ for which the corresponding principal series is not irreducible. If $\nu_0 = 0$ then it can be proved that there is no complementary series (th. 6.1), i.e., that $J_{\xi,\nu}$ is not unitary for any value of $\nu$.

Otherwise by means of the Dirac operator we determine a value $\nu^*$ of $\nu$, $\nu^* > 0$, for which $J_{\xi,\nu}$ is not unitary outside the interval $(0, \nu^*)$. If $\nu_0 \leqslant \nu^* < \nu_1$ then by a well known argument (lemma 1.3) it can be shown that the corresponding Langlands quotient is unitary iff $0 < \nu \leqslant \nu_0$.

More precisely it is unitary iff is either at the end or in the complementary series determined by such a $\xi$. If $\nu^*$ is not in this interval then we have that $\nu^* = \nu_1$.

We shall investigate this case in §5, the main result being that all such representations $J_{\xi,\nu_1}$ are unitary isolated (Cor. 5.4).

Essentially what is proved is that the kernel of the intertwining operator at $\nu_0$ is "contained" (in the kernel of the intertwining operator at $\nu_1$ and that any $K$-type has either 0 or "full multiplicity in $J_{\xi,\nu_0}$" (cf. th. 5.2 for a precise statement).

To reach this conclusion it is necessary to know not only whether the principal series representation is irreducible or not, but precisely its composition factors and to have the possibility of explicitly computing the $K$-multiplicities.

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