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EXISTENCE OF L^2-INTEGRABLE HOLOMORPHIC FORMS AND LOWER ESTIMATES OF T_V^1

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§0. **Introduction.** Let (V,q) be normal isolated singularity of dimension $n \geqslant 2$. It is easy to see that holomorphic functions defined on $V - \{q\}$ can be extended across q. However for holomorphic forms, the situation is completely different. Even if we assume that the holomorphic form ω defined on $V - \{q\}$ is L^2-integrable in a neighborhood of q in the sense of Griffiths ([4], [13]), it is not clear whether ω can be extended across q. In [13], the Griffiths number $g^{(p)}$ was introduced to measure how many L^2-integrable holomorphic forms on $V - \{q\}$ cannot be extended across q. Similarly, let us denote the number of holomorphic p-forms on $V - \{q\}$ which cannot be extended across q by $\delta^{(p)}$. In case of hypersurface singularities, these invariants are computed (cf. Theorem 1.1). One can see among these numbers, $g^{(n)}$, $g^{(n-1)}$, $\delta^{(n)}$ and $\delta^{(n-1)}$ are the most interesting invariants. The following are our main theorems:

THEOREM A. *Let $\pi : M \to V$ be any resolution of the singularity of V. Then*
(a) *For $n = 2$, $g^{(2)} \geqslant 1$ and $\delta^{(2)} \geqslant 1 + \dim H^1(M, \mathcal{O})$*
(b) *For $n \geqslant 3$, $g^{(n)} \geqslant n - 1$ and $\delta^{(n)} \geqslant n - 1 + \dim H^{n-1}(M, \mathcal{O})$ if $\dim H^{n-1}(M, \mathcal{O}) > 0$.*

THEOREM B. *Suppose that (V,q) admits a $\mathbf{C}^*$-action. Then $g^{(n-1)} \geqslant g^{(n)}$ and $\delta^{(n-1)} \geqslant \delta^{(n)}$.*

The invariant $\delta^{(n-1)}$ is of particular interest because in the case of Gorenstein surface singularity, $\delta^{(n-1)}$ is exactly equal to $\dim T_V^1$ where T_V^1 is the set of isomorphism class of first order infinitesimal deformation of V. By Grauert [3], T_V^1 can be thought of as a Zariski tangent space of the moduli space of (V,q). An important application of Theorem A and Theorem B is the following corollary.

COROLLARY C. *Let (V,q) be a Gorenstein surface singularity with C^*-action. Then $\dim T_V^1 \geqslant 1 + \dim H^1(M, \mathcal{O})$.*

We should remark that it is a long standing conjecture that $\dim T_V^1 > 0$ for Gorenstein surface singularities. In case (V,q) admits a C^*-action, our Corollary C is far better than the original conjecture. In fact in [14], we have already proved that $\dim T_V^1 > 0$ for Gorenstein surface singularities with C^*-action. More recently J. Wahl has informed us that he has obtained $\dim T_V^1 > 0$ for two-dimensional normal singularities with C^*-action. The detail of his proof will be available soon.

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