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DIRECTIONAL DERIVATIVE OF THE INCREASING REARRANGEMENT MAPPING AND APPLICATION TO A QUEER DIFFERENTIAL EQUATION IN PLASMA PHYSICS

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Introduction. In this paper we give some properties of the rearrangement of functions which do not seem to be known in the literature. In particular, we are interested in finding the directional derivatives of the mapping $u \to u^*$ from $L^2(\Omega)$ into $L^2(\Omega^*)$, where Ω is an open bounded set of R^N, $\Omega^* = (0, \operatorname{meas} \Omega)$, $u \in L^2(\Omega)$ and $u^* \in L^2(\Omega^*)$ is its increasing rearranged function. We also transform the integrals on Ω^* which involve rearranged functions, into integrals on Ω which involve the original ones. Doing this, we introduce new mean value operators which have very nice properties, and appear to be useful when applied to problems of the Grad-Mercier type which arise in Plasma physics. Specifically, we show that, in some sense, an equation

$$\begin{cases} -\Delta u + g(\underline{\beta}_u, u) = f \quad \text{in} \quad \Omega, \\ u = 0 \quad \text{on} \quad \partial\Omega, \\ \underline{\beta}_u(x) = \operatorname{meas}\{ y \in \Omega \mid u(y) < u(x) \} \end{cases}$$

is variational on the Sobolev space $H_0^1(\Omega)$, i.e., is the Euler equation of a variational problem on $H_0^1(\Omega)$ involving u and u^*. This work is a step in the understanding of more complicated variational problems presented in [11].

The plan is the following:

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