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# ON ZETA FUNCTIONS OF MATRIX ALGEBRAS AND DISTRIBUTIONS

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**Introduction.** Let $\Pi$ be an irreducible, smooth representation of $G = \mathrm{GL}_n(F)$, where $F$ is a nonarchimedean, local field. Denote the category of such representations by $\operatorname{Irr} G$. If $s \in \mathbf{C}$, $\operatorname{Re} s \gg 0$, and $\Phi$ is a Schwartz–Bruhat function on the matrix algebra $M_n = M_n(F)$, then the following integral is convergent:

$$Z(\Phi, \Pi, s) = \int_G \Phi(x)\Pi(x)\alpha^s(x)\, d^\times x \qquad \text{where} \quad \alpha^s(x) = |\det x|^s$$

By results of Godement and Jacquet in (G. J.), $Z(\Phi, \Pi, s)$ can be extended as a meromorphic, operator valued function of $s$. The operator valued distribution $\Phi \to Z(\Phi, \Pi, s)$ is called the Zeta distribution of $M_n$. Its behavior under the left and right multiplicative translations (denoted by $\lambda$ and $\rho$) is described by the following equations:

$$\begin{aligned} \lambda(g)Z(\Pi,s) &= \Pi(g^{-1})\alpha^{-s}(g)Z(\Pi,s) \qquad g \in G \\ \rho(g)Z(\Pi,s) &= Z(\Pi,s)\Pi(g)\alpha^s(g). \end{aligned} \tag{t}$$

If $s$ is a pole of the Zeta distribution, then the first coefficient of the Laurent expansion of $Z$ has the same invariance properties. Let $J_{\pi\cdot\alpha^s}$ denote these operator valued distributions which satisfy ($t$).

A. Weil has proved that $\dim J_\Pi = 1$ if $n = 1$(W1) or $\Pi$ is cuspidal (J).

The main result of this paper is the following:

THEOREM 3. *If* $\Pi \in \operatorname{Irr} G$ *and* $\Pi$ *is generic then* $\dim J_\Pi = 1$.

By a generic representation we mean one which admits a Whittaker model. It is a "nondegenerate representation" in the terminology of (BZ). All tempered irreducible representations are generic.

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**Preliminaries.** We use results of Bernstein and Zelevinski throughout this paper and we adopt their notation.

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