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HOMOLOGY OF SL(n, Z[$1/p$])

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The primary aim of this paper is to construct a CW complex Y suitable for the study of $H_*(SL(n,\mathbf{Z}[1/p]))$. In particular, the complex is to be of dimension $(n+2)(n-1)/2$, the virtual cohomological dimension of $SL(n,\mathbf{Z}[1/p])$, and the group is to act cellularly, properly, and with compact quotient. The homology of the group may then be found by means of cellular homology.

$SL(n,\mathbf{Z}[1/p])$ is a discrete subgroup of $SL(n,\mathbf{R}) \times SL(n,\mathbf{Q}_p)$. Therefore, the complex Y should reflect both the embedding in the real Lie group and the embedding in the p-adic group. Let X be the space of real positive definite quadratic forms modulo scalars. There is a natural action of $SL(n,\mathbf{R})$ on X and, with a suitable bordification, X has a polyhedral decomposition given by Voronoi [15]. To represent the p-adic embedding, let B be the Bruhat-Tits building associated to $SL(n,\mathbf{Q}_p)$. Y will be constructed as a subspace of $X \times B$.

Finally, a relationship between the polyhedral structures of X and B will be employed to compute the Euler characteristics of certain subgroups of $SL(3,\mathbf{Z})$. From these calculations we may conclude that $\chi(SL(3,\mathbf{Z}[1/p])) = 0$ for all primes p.

This paper, with some small modifications, is my Ph.D. dissertation. I wish to thank Ken Brown for his help and constant encouragement in the writing of the thesis.

§1. We begin by describing the reduction theory of real positive definite quadratic forms of Voronoi [15]. Lee and Szczarba used Voronoi's theory to produce an $SL(n,\mathbf{Z})$-equivariant CW complex which allowed them to compute the free summand of the cohomology of $SL(n,\mathbf{Z})$. A deformation retraction of the CW complex will also be introduced to obtain a lower dimensional complex which will be used in the construction of an $SL(n,\mathbf{Z}[1/p])$-equivariant CW complex.

Fix a positive integer n and let C be the cone of positive definite symmetric $n \times n$ matrices with coefficients in $\mathbf{R}$. C may be identified with the space of positive definite quadratic forms in n variables via $q(v) = v^tAv$.

A symmetric matrix is said to have rational nullspace if the kernel of the associated bilinear form has a basis consisting of vectors in $\mathbf{Q}^n$. Define $\overline{C}$ to be the space of non-zero symmetric $n \times n$ positive semi-definite matrices with rational nullspace. $GL(n,\mathbf{Q})$ will act from the right on $\overline{C}$ via $A \cdot g = g^tAg$.

Define a lattice in $\mathbf{Q}^n$ to be a subgroup which is isomorphic to $\mathbf{Z}^n$. For any

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