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HOLOMORPHIC FUNCTIONS OF BOUNDED MEAN OSCILLATION AND MAPPING PROPERTIES OF THE SZEGÖ PROJECTION

STEVEN G. KRANTZ

Dedicated to E. Stein

0. Let $B = \{z \in \mathbf{C}^n : |z| < 1\}$. If $0 < r < 1$, $P \in \partial B$, define

$$\beta_1(P,r) = \{\zeta \in \partial B : |\zeta - P| < r\}$$

$$\beta_2(P,r) = \{\zeta \in \partial B : |1 - \zeta \cdot \bar{P}| < r\}.$$

The balls β_1 are modelled on the singularity of the Poisson kernel and are suitable for classical "real variable" potential theory. The balls β_2 are modelled on the singularity of the Szegö kernel and are suitable for the potential theory of several complex variables.

We have two corresponding spaces of functions of bounded mean oscillation:

$$\mathrm{BMO}_j(\partial B) = \left\{ f \in L^1(\partial B) : \sup_{\substack{P \in \partial B \\ 0<r<1}} \int_{\beta_j(P,r)} |f(\zeta) - f_{\beta_j(P,r)}| \, d\sigma(\zeta)/\sigma(\beta_j(P,r)) \right.$$

$$\left. \equiv \|f\|_{\mathrm{BMO}_j} < \infty \right\}, \qquad j = 1,2.$$

Here $d\sigma$ is rotationally invariant area measure (Hausdorff measure) on ∂B, and $f_S \equiv \int f(\zeta) d\sigma(\zeta)/\sigma(S)$ for any measurable set $S \subseteq \partial B$ of positive σ measure.

The importance of BMO in analysis is well established (see [8], [10]), so we shall not discuss it. For $0 < p < \infty$, let

$$\mathcal{H}^p(B) = \left\{ f \quad \text{holomorphic on } B\text{: } \sup_{0<r<1} \int_{\partial B} |f(r\zeta)|^p \, d\sigma(\zeta) \equiv \|f\|^p_{\mathcal{H}^p(B)} < \infty \right\}$$

It is known (see [23] and references therein) that elements $f \in \mathcal{H}^p(B)$ have radial boundary values, indeed admissible boundary values, a.e. $[d\sigma]$. More precisely, if $\alpha > 0$, $P \in \partial B$, define the non-tangential approach region and the admissible approach region respectively by

$$\Gamma_\alpha(P) = \{z \in B : |z - P| < \alpha(1 - |z|)\}$$

$$\mathcal{Q}_\alpha(P) = \{z \in B : |1 - z \cdot \bar{P}| < \alpha(1 - |z|)\}.$$

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