Vol. 45, No. 3 DUKE MATHEMATICAL JOURNAL© September 1978

TOPOLOGICAL SINGULAR CIRCLE FIBERINGS

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1. Introduction

A singular circle fibering (SCF) is an open and closed surjective map $\pi : X \to Y$ of topological spaces, with the property that each point inverse $\pi^{-1}(y)$ is a circle or a point and some $\pi^{-1}(y)$ is a circle. Orbit maps of circle actions, Montgomery-Samelson singular fiberings with fiber S^1 [12], Seifert fiber spaces with fiber S^1 in the sense of Holmann [9] or Conner and Raymond [5], and stable circle foliations all determine SCF's. A basic question which we consider is whether a given SCF $\pi : X \to Y$ is an *orbit map*, i.e., whether there exists an S^1-action on X covering the identity on Y such that the induced map $X/S^1 \to Y$ is a homeomorphism. Define an SCF $\pi : X \to Y$ to be *local action* if each $y \in Y$ has a neighborhood U such that $\pi|\pi^{-1}(U)$ is an orbit map. Then our previous question naturally splits into two parts, to characterize those local actions which are orbit maps and to characterize the SCF's which are local actions. In this paper we answer these questions for a large class of SCF's including those with domain an integral cohomology manifold. Also we study the cohomology properties of SCF's, and derive some "Smith-type" theorems.

In Section 2 we consider the problem of determining when a local action $\pi : X \to Y$ is an orbit map. Let

$$F = \{x \in M | \pi^{-1}\pi(x) = \{x\}\}.$$

We denote an action of $S^1 = \mathbb{R}/\mathbb{Z}$ on X by a map $\phi : S^1 \times X \to X$. If $\alpha \in \operatorname{Aut} S^1 = \{\pm 1\}$, then $\phi\alpha$ denotes the action $\phi \cdot (\alpha \times 1_X)$. Any S^1-action considered on a space X will be assumed to be effective on each component of X.

Define an S^1-action on a space X to be *strongly effective* if the union of the principal orbits (those with trivial isotropy group) is dense in X. This is always true of an S^1-action on a cohomology manifold over $\mathbb{Z}$, [11]. Similarly a local action π is strongly effective if each fiber has a neighborhood on which π is the orbit map for a strongly effective action.

Our first main result (Theorem 2.8) is a uniqueness theorem for circle actions with the same underlying SCF. If $\pi : X \to Y$ is an SCF with X a paracompact Hausdorff space and ϕ and ψ are S^1-actions on X with π as orbit map such that ϕ and ψ induce the same orientation on each circle fiber of π, and if either $X - F$ is connected or ϕ and ψ are strongly effective, then ϕ and ψ are equiva-

Received September 20, 1977. Revision received April 7, 1978. Both authors were partially supported by National Science Foundation grants.