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TOTALLY GEODESIC SUBMANIFOLDS OF SYMMETRIC SPACES, II

BANG-YEN CHEN AND TADASHI NAGANO

§1. Introduction

An isometric immersion $f: B \to M$ of a Riemannian manifold B into another Riemannian manifold M is called *totally geodesic* if the geodesics in B are carried into geodesics in M. Given a Riemannian manifold M, it is fundamental and interesting to find out all Riemannian manifolds which admit totally geodesic immersions into M. In the first part of this series, totally geodesic submanifolds in the complex quadratic hypersurface $S0(m + 2)/S0(2) \times S0(m)$ have been completely classified.

In this part of this series we will introduce a new method to study symmetric spaces and their totally geodesic submanifolds. By using our new method, totally geodesic submanifolds in symmetric spaces of rank one can be obtained in a very easy way. Totally geodesic submanifolds in symmetric spaces of rank two will be given in the last section by applying our method. Basically, our method works as follows. For each compact symmetric space M and each closed geodesic c in M through the origin 0, we denote by p the antipodal point of 0 on c. We attach to p a pair of totally geodesic submanifolds of M, namely $(M_+(p), M_-(p))$. If B is a complete totally geodesic submanifold of M, then for each pair (B_+, B_-) for B, there exists a pair (M_+, M_-) for M such that B_+ and B_- are totally geodesic submanifolds of M_+, M_- respectively. Two pairs $(M_+(p), M_-(p))$ and $(M_+(p'), M_-(p'))$ are identified if there exists an isometry s which carries $0, p, M_+(p), M_-(p)$ into $0, p', M_+(p'), M_-(p')$, respectively. We denote by $P(M)$ the set of such pairs. Then for any totally geodesic submanifold B in M, we have an induced mapping from $P(B)$ to $P(M)$, which is called the pairwise totally geodesic immersion. If B and M have the same rank, the pairwise totally geodesic immersion is surjective, and it is bijective when B and M have the same Weyl group. Using these we may classify totally geodesic submanifolds of symmetric spaces.

In §2 we will develop a general theory for the pairs. Relations between totally geodesic submanifolds, Weyl groups and root systems of symmetric spaces will be obtained. In §3, we will discuss how the pairs $P(M)$ are related to the corresponding pairs of any other locally isometric space. Complete relations will be given. In §4, the pairs for group manifolds are discussed in detail. Complete lists

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