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THE SZEGÖ KERNEL IN TERMS OF CAUCHY-FANTAPPIÈ KERNELS

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Contents

Introduction

A. The Szegö kernel and the Henkin-Ramirez kernel.

Two chief methods to represent a holomorphic function u in $\Omega \subset \mathbb{C}^n$ in terms of its boundary values on $b\Omega$ are the Szegö kernel $S(w, z)$, [11], [19] and the Henkin-Ramirez kernel $H(w, z)$, [7], [18], and we will show that S can be written in terms of H.

It is assumed throughout that Ω is strictly pseudoconvex, smooth $\subset\subset \mathbb{C}^n$, since this is the case for which H has been constructed. Now S is very easy to define and gives the *orthogonal* projection $\mathbf{S} : L^2(b\Omega) \to \mathcal{H}^2(b\Omega)$ on the subspace of boundary values of holomorphic functions (See §3)

$$\mathbf{S}u(w) = \int_{z \in b\Omega} S(w, z)u(z)d\sigma_z \qquad w \in \Omega \tag{1}$$

where $d\sigma_z$ is Lebesgue area of $b\Omega$.

But it is much more difficult to find out non-trivial properties of S. Only recently, Fefferman [4] and later Boutet de Monvel and Sjöstrand [3] have completely determined the main term of the singularity of $S(w, z)$ at $w = z$. It is not clear at present which coefficients in their asymptotic expansions vanish, e.g., in particular, for which Ω logarithmic terms appear or don't.

On the other hand, H is highly explicit, non canonical (choices are involved in its construction), and its singularity is precisely the main term of S mentioned above. But it is hopeless to try to adjust the construction of H so that $H = S$. No logarithmic terms could ever appear if such an adjustment were possible, and they are known to be present for certain Ω.

The methods of [4] and [3] rely essentially on Kohn's estimates for the $\bar{\partial}$-Neumann problem, while ours are, in a sense, more simple-minded.

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