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AN AUXILIARY RESULT IN THE THEORY OF TRANSCENDENTAL NUMBERS II

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1. Introduction. In an earlier paper [6] on the same subject, the second author of the present paper considered so-called exponential polynomials, that is, functions of the type

$$F(z) = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} A_{km} z^m e^{\omega_k z}, \tag{1}$$

with complex numbers A_{km} and ω_k. Some theorems in that paper have the following structure:

If S and T are natural numbers, such that ST is large enough, say $ST \geq a(K, M, \omega_0, \omega_1, \cdots, \omega_{K-1})$ and if E denotes the maximal absolute value of all numbers $F^{(t)}(s)$ for $s = 0, 1, \cdots, S-1$ and $t = 0, 1, \cdots, T-1$, then

$$\max_{k,m} |A_{km}| \leq b(K, M, \omega_0, \omega_1, \cdots, \omega_{K-1})E. \tag{2}$$

Here a and b are explicitly given expressions in K, M, $\omega_0, \omega_1, \cdots, \omega_{K-1}$.

These results have been used by several authors; for instance by Ramachandra and Shorey [5] and by the first author of this paper [3]. A great advantage for applications is the rather simple form of (2) and of the expressions a and b. However, in some respect the paper [6] was out-dated already at the time of its publication. Its theorems were supposed to be useful in proofs of non-trivial lower bounds for linear forms in logarithms of algebraic numbers. But the lower bounds given by Fel'dman [4] and Baker in recent papers [e.g. 1] and which are very sharp with respect to some of the variables, cannot be proved by means of any theorem from [6].

The main purpose of the present paper is, to give a special variant of these theorems which does not have this disadvantage. Essentially, this will be done by replacing in the inequality (2) A_{km} by $m!\, A_{km}$ and $E = \max |F^{(t)}(s)|$ by $\max 1/t!\ |F^{(t)}(s)|$. Further, we give some lemmas which make it easier to apply our results. An application of the main result of this paper to transcendental number theory will be given by the first author in a subsequent paper.

2. The main theorem. In this section we make the following assumptions and use the following notation:

F_1 denotes the exponential polynomial

$$F_1(z) = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} \frac{B_{km}}{m!} z^m e^{\omega_k z} \tag{3}$$

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