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A TORSION VERSION OF THE CHASE-ROSENBERG EXACT SEQUENCE

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Suppose R is a commutative ring and S is a commutative ring extension of R which is finitely generated, faithful and projective as an R-module. Chase and Rosenberg [CR] have constructed an exact sequence

$$0 \to H^1(S/R, U) \to \text{Pic}(R) \to H^0(S/R, \text{Pic}) \to H^2(S/R, U)$$
$$\to B(S/R) \to H^1(S/R, \text{Pic}) \to H^3(S/R, U)$$

where U is the units functor, Pic (R) is the Picard group of invertible R-modules, $B(S/R)$ is the relative Brauer group and the cohomology is that of Amitsur. Their sequence generalized the classical Theorem 90 of Hilbert and the isomorphism of the relative Brauer group of a finite Galois field extension with the second Galois cohomology group.

The main tools used by Chase and Rosenberg were spectral sequences and limits over Zariski covers. We obtain a similar sequence by employing non-abelian Amitsur cohomology and limits over split Azumaya algebras.

We begin by outlining non-abelian Amitsur cohomology theory. We then show how to modify certain admissible categories of structured modules so as to have limits over them preserve exactness. Next we show that in certain cases, the modified categories are equivalent to the category of split Azumaya algebras and we construct certain functor sequences related to the Skolem–Noether Theorem. Lastly, we produce our six term sequence by applying the cohomology theory of section one and the limit theory of section two to the functor sequences of section three.

A word about notation. R is always a commutative ring with 1. Unless otherwise specified, all modules, algebras, tensor products, hom sets, structured categories, etc., are defined over R.

1. Non-abelian Amitsur cohomology. $\mathbf{M}$ is the category of (left) R-modules. $\mathbf{M}_f$ and $\mathbf{M}_p$ are the full subcategories of respectively faithfully flat and faithfully projective (i.e., finitely generated, faithful and projective) modules. $\mathbf{K}$ is the category of commutative R-algebras. $\mathbf{K}_f$ and $\mathbf{K}_p$ are the full subcategories of commutative algebras whose underlying module structures lie in respectively $\mathbf{M}_f$ and $\mathbf{M}_p$.

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